Temporalizing Epistemic Default Logic

W. van der Hoek, J.-J. Meyer, and J. Treur

UU-CS-1994-54
December 1994

Utrecht University

Department of Computer Science

Padualaan 14, P.O. Box 80.089,
3508 TB Utrecht, The Netherlands,
Tel. : ... + 31 - 30 - 531454

Temporalizing Epistemic Default Logic

W. van der Hoek, J.-J. Meyer, and J. Treur

Technical Report UU-CS-1994-54
December 1994

Department of Computer Science
Utrecht University
P.O.Box 80.089
3508 TB Utrecht
The Netherlands

ISSN: 0924-3275

Temporalizing Epistemic Default Logic[+]

Wiebe van der Hoek[a]
John-Jules Meyer[b]
Jan Treur[c]

1. Introduction

In [MH93a,b] a nonmonotonic logic was introduced, called Epistemic Default Logic (**EDL**). It is based on the metaphor of a meta-level architecture. It already has been established in [MH93a,b] how upward reflection can be formalized by a nonmonotonic entailment based on epistemic states, and the meta-level process by a (monotonic) epistemic logic. The meta-level reasoning can be viewed as the part of the reasoning pattern where it is determined what the possibilities are for default assumptions to be made, based on which information is available and (especially) which is not. The outcome at the meta-level concerns conclusions of the form $P\varphi$, where φ is an object-level formula. In EDL, default conclusions are kept separate from the object level knowledge (they remain at the meta-level), by means of this explicit default operator P (just like in NML3, see [Doh91]). If one wants to draw further conclusions from them using object level knowledge this should be done at the meta-level. Compared to a meta-level architecture, what was still missing was the step where the default assumptions are actually made, i.e., where such formulas φ are added to the object level knowledge, in order to be able to reason further with them at the object level. Here we actually "jump (down) to conclusions". This is what should be achieved by the downward reflection step. In the current paper we will introduce a formalization of the downward reflection step in the reasoning pattern as well. Thus a formalization is obtained of the reasoning pattern as a whole consisting of a process of generating possible default assumptions and actually assuming them (a similar pattern as generated by the so-called BMS-architecture introduced in [TT91]).

The formalization of downward reflection is inspired by [Tre94, HMT94], where it is pointed out how temporal models can provide an adequate semantics for meta-level architectures in general, and [ET93, ET94] where these ideas have been worked out to obtain a linear time temporal semantics for default logic. The general idea is that conclusions derived at the meta-

[+] This work is partially supported by ESPRIT III Basic Research Action No. 6156 (DRUMS II).
[a] Utrecht University, Dept. of Computer Science, P.O. Box 80.089, 3508 TB Utrecht. Email: wiebe@cs.ruu.nl. Also partially supported by the Free University of Amsterdam.
[b] Like footnote a, but also partially supported by the University of Nijmegen. Email: jj@cs.ruu.nl.
[c] Free University of Amsterdam, Dept. of Mathematics and Computer Science, De Boelelaan 1081a, 1081 HV Amsterdam, The Netherlands. Email: treur@cs.vu.nl.

level essentially are statements about the state of the object level reasoning at the next moment of time. So we can define downward reflection as a shift of time in a (reasoning) process that is described by temporal logic.

In this paper in the Sections 2 to 6 the logic EDL is presented. As compared with earlier publications ([MH91a, MH92a, MH93a,b]), the logic is slightly extended in order to cater for downward reflection in subsequent sections. In Section 7 we define a labeled branching time temporalization of this logic, in spirit of the approach of [FG92]. In Section 8 we define sceptical and credulous entailment relations based on temporal models.

2. Epistemic logic

2.1. DEFINITION (epistemic formulas). We first introduce the language of epistemic logic that we shall consider in the first instance. Let **P** be a set of propositional constants (atoms); $\mathbf{P} = \{p_k \mid k \in I\}$, where I is either a finite or countably infinite set. The set FORM of *epistemic formulas* φ, ψ,... is the smallest set containing **P**, closed under the classical propositional connectives and the epistemic operator K, where $K\varphi$ means that φ is known. Moreover, we use $M\varphi$ as an abbreviation for $\neg K \neg\varphi$, meaning that φ is epistemically possible. An *objective formula* is an epistemic formula without any occurrences of the modal operators K and M. For $\Gamma \subseteq$ FORM, we denote by Prop(Γ) the set of objective formulas in Γ.

Objective formulas are interpreted on ordinary valuations:

2.2. DEFINITION (propositional models). A *valuation* is a function from **P** to $\{\mathbb{t}, \mathbb{f}\}$. The set of all valuations is denoted $\mathbb{W}$. The powerset of $\mathbb{W}$ is denoted by ModSet(**PC**). For any M $\in$ ModSet(**PC**) and each objective formula φ we define M $\models \varphi$ iff each w $\in$ M is a valuation for φ.

To interpret the whole set FORM of epistemic formulas, we need richer structures:

2.3. DEFINITION (S5-Kripke models). A (simple) S5-model is a structure $\mathbb{M} = \langle M, \pi_{\mathbb{M}}, R_{\mathbb{M}} \rangle$ where M is a non-empty set, the elements of which are called worlds, $\pi_{\mathbb{M}}$ is a truth assignment function of type $M \rightarrow (\mathbf{P} \rightarrow \{\mathbb{t}, \mathbb{f}\})$ such that for all $m_1, m_2 \in M$: $\pi_{\mathbb{M}}(m_1) = \pi_{\mathbb{M}}(m_2) \Rightarrow m_1 = m_2$, and $R_{\mathbb{M}}$ is the universal accessibility relation on M, i.e., $R_{\mathbb{M}} = M \times M$. The class of (simple) S5-models is denoted by Mod(**S5**).

The more general definition of an S5-model requires $R_{\mathbb{M}}$ only to be an equivalence relation, but for our purposes we will assume S5-models always to be simple in the sense that we defined above. One can show that this does not change the validities of the logic (cf. Def. 2.6 below), but it facilitates a number of technical issues such as the definition of submodels and union of models (cf. Def. 2.4 below). However, some caution is in order: although the restriction to simple S5-models does not affect the notion of validity, some of the propositions on S5-models below *do* depend on the simplicity of the models, and do *not* hold for general S5-models.

The set of worlds in an S5-model represents a collection of alternative worlds that are considered (equally) possible on the basis of (lack of) knowledge. In the next section we shall use S5-models as representations of the reasoner's objective knowledge and epistemic meta-knowledge (i.e., what he knows that he knows or does not know).

Note that, for any $m \in M$, the function $\pi_{\mathbb{M}}(m) = \lambda p \cdot \pi_{\mathbb{M}}(m)(p)$ is a valuation. Since we have that in an S5-model it holds that $\pi_{\mathbb{M}}(m_1) = \pi_{\mathbb{M}}(m_2) \Leftrightarrow m_1 = m_2$, we may identify worlds m with their valuations $\pi_{\mathbb{M}}(m)$, and write, for $m \in M$, $m \equiv \pi_{\mathbb{M}}(m) = \lambda p \cdot \pi_{\mathbb{M}}(m)(p)$. So, without loss of generality, we may consider S5-models of the form $\mathbb{M} = \langle M, \pi_{\mathbb{M}}, R_{\mathbb{M}} \rangle$ with $M \subseteq \mathbb{W}$, and in the sequel we will assume this indeed to be the case.

Two S5-models $\mathbb{M}_1 = \langle M_1, \pi_1, R_1 \rangle$ and $\mathbb{M}_2 = \langle M_2, \pi_2, R_2 \rangle$ are called *compatible* if for every m $\in M_1 \cap M_2$ it holds that $\pi_1(m) = \pi_2(m)$. Note that models in which worlds are identified with their valuation functions (and thus are such that $M \subseteq \mathbb{W}$) are always compatible.

2.4. DEFINITION (submodels and union of S5-models). We define a subset relation on compatible S5-models by: $\mathbb{M}_1 \subseteq \mathbb{M}_2$ iff $M_1 \subseteq M_2$. Moreover, if $\mathbb{M}_1 = \langle M_1, \pi_1, R_1 \rangle$ and $\mathbb{M}_2 = \langle M_2, \pi_2, R_2 \rangle$ are two compatible S5-models, their union is defined as: $\mathbb{M}_1 \cup \mathbb{M}_2 = \langle M, \pi, R \rangle$, where $M = M_1 \cup M_2$, $\pi(m) = \pi_i(m)$ if $m \in M_i$ $(i = 1, 2)$, and $R = M \times M$.

Note that π is well-defined because of the compatibility of $\mathbb{M}_1$ and $\mathbb{M}_2$.

2.5. DEFINITION (interpretation of epistemic formulas). Given $\mathbb{M} = \langle M, \pi_{\mathbb{M}}, R_{\mathbb{M}} \rangle$, we define the relation $(\mathbb{M}, m) \models \varphi$ by induction on the structure of the epistemic formula φ:

$$
\begin{array}{lll}
(\mathbb{M}, m) \models p & \Leftrightarrow & \pi_{\mathbb{M}}(m)(p) = t \text{ for } p \in \mathbf{P} \\
(\mathbb{M}, m) \models \psi_1 \wedge \psi_2 & \Leftrightarrow & (\mathbb{M}, m) \models \psi_1 \text{ and } (\mathbb{M}, m) \models \psi_2 \\
(\mathbb{M}, m) \models \neg\psi & \Leftrightarrow & (\mathbb{M}, m) \not\models \psi \\
(\mathbb{M}, m) \models K\psi & \Leftrightarrow & (\mathbb{M}, m') \models \psi \text{ for all } m' \text{ such that } R_{\mathbb{M}}(m, m') \\
(\mathbb{M}, m) \models M\psi & \Leftrightarrow & (\mathbb{M}, m') \models \psi \text{ for some } m' \text{ such that } R_{\mathbb{M}}(m, m').
\end{array}
$$

Note that, in the present setting, the clause for the *K*-operator amounts to $(\mathbb{M}, m) \models K\psi$ being true iff $(\mathbb{M}, m') \models \psi$ for all $m' \in M$. Thus, it states that φ is known precisely when φ holds in the whole set of epistemic alternatives; that for the M-operator states that φ is considered epistemically possible iff there is at least one epistemic alternative that satisfies φ.

2.6. DEFINITION (validity and satisfiability).

(i) φ is *valid in an S5-model* $\mathbb{M} = \langle M, \pi_{\mathbb{M}}, R_{\mathbb{M}} \rangle$, denoted $\mathbb{M} \models \varphi$, if for all $m \in M$: $(\mathbb{M}, m) \models \varphi$.

(ii) φ is *valid*, notation $\text{Mod}(\mathbf{S5}) \models \varphi$, if $\mathbb{M} \models \varphi$ for all S5-models $\mathbb{M}$.

(iii) φ is *satisfiable* if there is an S5-model $\mathbb{M} = \langle M, \pi_{\mathbb{M}}, R_{\mathbb{M}} \rangle$, and a world $m \in M$ such that $(\mathbb{M}, m) \models \varphi$.

Validity w.r.t. S5-models can be axiomatized by the system **S5**:

2.7. DEFINITION (system **S5**). The logic **S5** consists of the following:

Axioms:

(A1)	All propositional tautologies	
(A2)	$(K\varphi \wedge K(\varphi \rightarrow \psi)) \rightarrow K\psi$	*Knowledge is closed under logical consequence.*
(A3)	$K\varphi \rightarrow \varphi$	*Known facts are true.*
(A4)	$K\varphi \rightarrow KK\varphi$	*One knows that one knows something.*
(A5)	$\neg K\varphi \rightarrow K\neg K\varphi$	*One knows that one does not know something.*

Derivation rules:

(R1) $\dfrac{\varphi \quad \varphi \rightarrow \psi}{\psi}$ *Modus Ponens*

(R2) $\dfrac{\varphi}{K\varphi}$ *Necessitation*

That φ is a theorem derived by the system **S5** is denoted by $\mathbf{S5} \vdash \varphi$.

2.8. THEOREM (Soundness and completeness of **S5**). $\mathbf{S5} \vdash \varphi \Leftrightarrow \text{Mod}(\mathbf{S5}) \models \varphi$

2.9. DEFINITION. We say a formula ψ is in *normal form* if it is a disjunction of conjunctions of the form $\delta = \alpha \wedge K\beta_1 \wedge K\beta_2 \wedge \ldots \wedge K\beta_n \wedge M\gamma_1 \wedge M\gamma_2 \wedge \ldots \wedge M\gamma_k$, where α, β_i and γ_j ($i \leq n$, $j \leq k$) are all objective formulas.

The following holds for **S5**-logic;

2.10. THEOREM ([MH94]) In **S5** every formula φ is equivalent to a formula ψ in normal form.

In particular, this theorem implies that every epistemic formula can be represented equivalently without nested epistemic modalities. In **S5** one thus can do without nestings.

3. Epistemic states and stable sets

In this paper we simply define an epistemic state as an S5-model. The idea behind this was already touched upon after Definition 2.3. The worlds in an S5-model represent the states of the (real) world that the reasoner considers possible. Thus the S5-model as a whole delimit the ways the real world is like as far as the reasoner is concerned. In other words, it determines what he knows about the world and what he does not (what he has doubts about by considering contradictory possibilities). Therefore, an S5-model represents truly the epistemic state of the reasoner.

3.1. DEFINITION. An *epistemic state* is an S5-model $\mathbb{M} = \langle M, \pi_{\mathbb{M}}, R_{\mathbb{M}} \rangle$. The set M is the set of epistemic alternatives allowed by the epistemic state $\mathbb{M}$.

3.2. DEFINITION. Let $\mathbb{M} = \langle M, \pi_{\mathbb{M}}, R_{\mathbb{M}} \rangle$ be an **S5**-Kripke model. Then $K(\mathbb{M})$ is the set of facts known in $\mathbb{M}$: $K(\mathbb{M}) = \{\varphi \mid \mathbb{M} \models K\varphi\}$. We call $K(\mathbb{M})$ the *theory of* $\mathbb{M}$ or *knowledge in* $\mathbb{M}$.

We mention here that the knowledge in $\mathbb{M}$ are exactly the validities in $\mathbb{M}$: $K(\mathbb{M}) = \{\varphi \mid \mathbb{M} \models K\varphi\} = \{\varphi \mid \mathbb{M} \models \varphi\}$. Cf. [MH94].

3.3. LEMMA. For any (simple) S5 models $\mathbb{M}_1$ and $\mathbb{M}_2$: $\mathbb{M}_1 \subseteq \mathbb{M}_2$ iff $Prop(K(\mathbb{M}_2)) \subseteq Prop(K(\mathbb{M}_1))$.

3.4. REMARK. The converse relation of $\subseteq$ on Kripke models (Cf. Definition 2.4), will play an important role in the sequel. $\mathbb{M}_1 \supseteq \mathbb{M}_2$ means that the model $\mathbb{M}_2$, viewed as a representation of the knowledge of a reasoner, involves a *refinement* of the knowledge associated with model $\mathbb{M}_1$. This has to be understood as follows: in the model $\mathbb{M}_2$ less (or the same) worlds are considered possible by the reasoner as compared by the model $\mathbb{M}_1$. So, in the former case the reasoner has less doubts about the true nature of the world. By Lemma 3.3 it turns out that this means that with respect to model $\mathbb{M}_2$ the reasoner has at least the objective knowledge associated with model $\mathbb{M}_1$, and possibly more. So in a transition of $\mathbb{M}_1$ to $\mathbb{M}_2$ we may say that objective knowledge is gained by the reasoner. Thus the relation '$\supseteq$' acts as an information ordering on the set of S5-models. Finally we remark that Lemma 3.3 is a typical example of a property that holds for simple S5-models only; with respect to general S5-models we would only have the 'only if' part of the Lemma (cf. the discussion following Def. 2.3).

3.5. PROPOSITION (Moore [Moo85]).

(i) The theory $\Sigma = K(\mathbb{M})$ of an epistemic state $\mathbb{M}$ is a so-called *stable set*, i.e., satisfies the following properties:

(St 1) all instances of propositional tautologies are elements of Σ;

(St 2) if $\varphi \in \Sigma$ and $\varphi \to \psi \in \Sigma$ then $\psi \in \Sigma$;

(St 3) $\varphi \in \Sigma \Leftrightarrow K\varphi \in \Sigma$

(St 4) $\varphi \notin \Sigma \Leftrightarrow \neg K\varphi \in \Sigma$

(St 5) Σ is propositionally consistent.

(ii) Every stable set Σ of epistemic formulas determines an **S5**-Kripke model $\mathbb{M}_\Sigma$ for which it holds that $\Sigma = K(\mathbb{M}_\Sigma)$. Moreover, if **P** is a finite set, then $\mathbb{M}_\Sigma$ is the unique **S5**-Kripke model with this property.

3.6. PROPOSITION. A stable set is uniquely determined by the objective formulas it contains.

3.7. REMARK. Thus stable sets act as the epistemic contents of an epistemic state (viz. an S5-model): a stable set $\Sigma = K(\mathbb{M})$ describes exactly the formulas known by the reasoner when he is is in epistemic state $\mathbb{M}$. From Prop. 3.5(i) we see that this knowledge is closed under classical propositional reasoning and under positive and negative introspection: if some formula is known also the fact that this is known is in its turn known by the reasoner, and if some formula is not known, the fact that it is not known is known itself. This reflects the (perhaps rather strong notion of) rationality of the reasoner. On the other hand, Prop. 3.5(ii) says that a stable set of epistemic formulas determines an S5-model, the epistemic state associated with (knowing) this stable set. Prop. 3.6 states that a stable set, and thus the associated epistemic state, is completely determined if one specifies exactly what *objective* knowledge is present at the reasoner. The rest of the known set of formulas (the stable set) then follows.

4. Entailment based on epistemic states: upward reflection

On the basis of epistemic states, Halpern & Moses define an entailment relation $\mid\!\sim$ with which one can infer what is known, and, more importantly, what is unknown in such epistemic states.

First we note that with every set of valuations we can associate an epistemic state, as follows:

4.1. DEFINITION. Given a set $M \subseteq \mathbb{W}$ of valuations, we define the *associated S5-model* $\Phi(M)$, given by $\Phi(M) = \langle M, \pi_M, R_M \rangle$ with π_M: $M \times \mathbf{P} \to \{\mathbb{t}, \mathbb{f}\}$ such that π_M: $(m, p) \mapsto m(p)$.

Intuitively, Φ associates with M the epistemic state given by considering (only) the set M of classical propositional models to be the possible states of the world: this set, so to speak, gives a partial description of the real world. In general, more than just one classical model is considered possible so that only those facts are known that are true in all of them. We can exploit this idea further and define an entailment relation based on the premise that we *only know* some objective formula φ. To this end we need some additional notation:

4.2. DEFINITION. Given some objective formula φ, we define M_φ as the set of valuations satisfying φ, i.e., $M_\varphi = \{m \in \mathbb{W} \mid m \models \varphi\}$. We denote the epistemic state $\Phi(M_\varphi)$ associated with M_φ by $\mathbb{M}_\varphi$.

We have the following alternative characterizations of $\mathbb{M}_\varphi$:

4.3. PROPOSITION. $\mathbb{M}_\varphi = \bigcup \{\mathbb{M} \mid \mathbb{M} \models \varphi\} = \bigcup \{\mathbb{M} \mid \mathbb{M} \models K\varphi\}$.

PROOF. The latter equality follows from the fact that we are in the realm of S5-models (cf. our remark following Def. 3.2). The former equality is proved as follows: $\mathbb{M}_\varphi = \Phi(M_\varphi) = \langle M_\varphi, \pi_{M_\varphi}, R_{M_\varphi}\rangle$ with $M_\varphi = \{m \in \mathbb{W} \mid m \models \varphi\}$. On the other hand, $\mathbb{M} = \langle M, \pi_M, R_M\rangle \models \varphi \Leftrightarrow$ (since φ is objective—adopting the view that $M \subseteq \mathbb{W}$) $M \models \varphi \Leftrightarrow M \subseteq M_\varphi$. So $\bigcup \{\mathbb{M} \mid \mathbb{M} \models \varphi\} = \bigcup \{\langle M, \pi_M, R_M\rangle \mid M \subseteq M_\varphi\} = \langle M_\varphi, \pi_{M_\varphi}, R_{M_\varphi}\rangle$, since $\bigcup \{M \mid M \subseteq M_\varphi\} = M_\varphi$. Thus $\mathbb{M}_\varphi = \bigcup \{\mathbb{M} \mid \mathbb{M} \models \varphi\}$. ■

Proposition 4.3 says that in order to get $\mathbb{M}_\varphi$, we can also consider all S5-models of φ and take their union to obtain one 'big' S5-model. We denote the mapping $\varphi \hookrightarrow \mathbb{M}_\varphi$ by μ: $\mu(\varphi) = \mathbb{M}_\varphi$.

Now we are ready to define an entailment relation based on this "big" model. Keeping in mind that '$\supseteq$' acts as an information order on models (Cf. 3.4), we define what are the consequences of knowing only the objective formula φ as those formulas that hold in the "big" model $\mathbb{M}_\varphi$:

4.4. DEFINITION (Nonmonotonic epistemic entailment). For $\varphi \in$ Prop(FORM), and $\psi \in$ FORM:

$$\varphi \mathrel{|\!\sim} \psi \Leftrightarrow \psi \in K(\mathbb{M}_\varphi).$$

Informally, this means that ψ is entailed by φ, if ψ is contained in the theory (knowledge) of the "largest S5-model" $\mathbb{M}_\varphi$ of φ. Halpern & Moses showed in [HM84] that this "largest model" need not always be a model of φ itself if we allow φ to contain epistemic operators. However, in our case where we only use objective formulas φ, $\mathbb{M}_\varphi$ is always the largest model for φ. This is obvious from our construction of $\mathbb{M}_\varphi$, since $M_\varphi \models \varphi$ and therefore $\mathbb{M}_\varphi = \Phi(M_\varphi) = \langle M_\varphi, \pi_{M_\varphi}, R_{M_\varphi}\rangle \models \varphi$ (since φ is objective). Moreover, in this case the theory $K(\mathbb{M}_\varphi)$ of this largest model is a stable set that contains φ and such that for all stable sets Σ containing φ it holds that Prop($K(\mathbb{M}_\varphi)$) $\subseteq$ Prop(Σ) (by Lemma 3.3 and Prop. 3.5), thus $K(\mathbb{M}_\varphi)$ is the "propositionally least" stable set that contains φ. So $\vdash$ can also be viewed as a *preferential entailment* relation in the sense of Shoham [Sho87, 88], where the preferred models of φ are the largest ones, viz. $\mathbb{M}_\varphi$, where the least objective knowledge is available.

We denote the mapping $\varphi \mapsto K(\mathbb{M}_\varphi)$ by κ: $\kappa(\varphi) = K(\mathbb{M}_\varphi)$, the stable set associated with knowing only φ. Alternatively viewed, $\kappa(\varphi)$ is the $\vdash$-closure of φ. Note that since $\kappa(\varphi) = K(\mathbb{M}_\varphi)$ is a stable set, it is also propositionally closed.

We give a few examples to show how the entailment $\vdash$ works: Let p and q be two distinct primitive propositions. Then:

$$p \vdash K(p \vee q)$$
$$p \vdash \neg K q$$
$$p \vdash Kp \wedge M\neg q$$
$$p \wedge q \vdash K(p \wedge q) \wedge Kp \wedge Kq$$
$$p \vee q \vdash K(p \vee q) \wedge M\neg p \wedge M\neg q$$

Obviously, the entailment relation $\vdash$ is nonmonotonic, justifying the name we have given it. (For instance, we have $p \vdash M\neg q$, while *not* $p \wedge q \vdash M\neg q$; it even holds that $p \wedge q \vdash \neg M\neg q$.)

The nonmonotonic epistemic entailment $\vdash$ enables us to derive from an objective formula φ, characterizing the exact epistemic state of the reasoner (viz., technically, the epistemic state $\mathbb{M}_\varphi$), exactly what is known and, even more importantly, *what is unknown* in this epistemic state. This latter property renders the entailment relation context-sensitive and nonmonotonic, so that the relation $\vdash$ goes beyond an entailment expressible in ordinary epistemic logic: with respect to its premise the relation $\vdash$ involves a kind of epistemic closure.

Finally we state a property of that we shall need in the sequel:

4.5. PROPOSITION. The entailment relation $\vdash$ enjoys the property:

$$\varphi \vdash \psi_1 \ \& \ \varphi \vdash \psi_2 \ \Rightarrow \ \varphi \vdash \psi_1 \wedge \psi_2 .$$

PROOF. Directly from Def. 4.4 and the fact that the stable set $K(\mathbb{M}_\varphi)$ is closed under conjunction (which in its turn follows from the fact that a stable set is closed under propositional reasoning and contains the tautology $\psi_1 \rightarrow (\psi_2 \rightarrow (\psi_1 \wedge \psi_2))$). ■

5. The epistemic preference logic S5P

The "upward reflection" entailment relation $\mid\!\sim$ enables us to derive information about what is known and what is not known. In this section we show how we can use this information to perform default reasoning. To this end we extend our language with operators that indicate that something is a *default belief* and thus has a different epistemic status than a *certain fact*. In this way the proverbial "jump to conclusions" is not made directly in the logic, but a somewhat more cautious approach is taken. The "jump" itself will be part of a next operation, the "downward reflection", which will be discussed in the next section when we incorporate a temporal element into our approach.

Let I be a finite set of indexes. The logic **S5P** of epistemic default logic is an extension of the epistemic logic **S5** by means of special modal operators P_i denoting *default belief* (w.r.t. situation or frame of mind i), for $i \in I$, and also generalisations P_τ, for $\tau \subseteq I$.

Informally, $P_i\varphi$ is read as "φ is a default belief (within frame of reference i)". As we shall see below, a frame of reference (or mind) refers to a preferred subset of the whole set S of epistemic alternatives. This operator is very close to the PA (possible assumption) of [TT91] and the D (default) operator of [Doh91]. The generalisation $P_\tau\varphi$ is then read as a default belief with respect to the (intersection of the) frames of reference occurring in τ.

Formally, S5P-formulas are interpreted on Kripke-structures (called S5P-models) of the form $\mathbb{M} = \langle M, \pi_{\mathbb{M}}, \{M_i \mid i \in I\}, R_{\mathbb{M}}, \{R_i \mid i \in I\}\rangle$, where M is a collection of worlds, $\pi_{\mathbb{M}}: M \times \mathbf{P} \to \{\mathbf{t}, \mathbf{f}\}$ is a truth assignment to the primitive propositions per world, $M_i \subseteq M$ ($i \in I$) are sets ('frames') of preferred worlds, $R_{\mathbb{M}} = M \times M$, and $R_i = M \times M_i$ ($i \in I$). When writing $\mathbb{M}_1 \subseteq \mathbb{M}_2$, we always mean that the set of worlds of $\mathbb{M}_1$ is a subset of those of $\mathbb{M}_2$. Again we may identify worlds s and their truth assignments $\pi(s)$. We let Mod(**S5P**) denote the collection of Kripke-structures of the above form. Given an S5P-model $\mathbb{M} = \langle M, \pi_{\mathbb{M}}, \{M_i \mid i \in I\}, R_{\mathbb{M}}, \{R_i \mid i \in I\}\rangle$, we call the S5-model $\mathbb{M}' = \langle M, \pi_{\mathbb{M}}, R_{\mathbb{M}}\rangle$ the *S5-reduct* of $\mathbb{M}$.

5.1. DEFINITION (interpretation of S5P-formulas). Given a model $\mathbb{M} = \langle M, \pi_{\mathbb{M}}, \{M_i \mid i \in I\}, R_{\mathbb{M}}, \{R_i \mid i \in I\}, \rangle$ we extend the truth definition on S5-models by the following clauses for the P_i- and P_τ-operators:

$(\mathbb{M}, m) \models P_i\varphi$ iff $(\mathbb{M}, m') \models \varphi$ for all m' with $R_i(m, m')$, for $i \in I$;
$(\mathbb{M}, m) \models P_\tau\varphi$ iff $(\mathbb{M}, m') \models \varphi$ for all m' with $R_\tau(m, m')$, where $R_\tau = \bigcap_{i \in \tau} R_i$ and $\tau \subseteq I$.

Thus the former clause states that $P_i\varphi$ is true if φ is a default belief w.r.t. subframe M_i, whereas the latter says that $P_\tau\varphi$ is true if φ is a default belief w.r.t. the intersection of the subframes M_i, $i \in \tau$. We will sometimes denote this intersection by M_τ. In the clause for P_τ, we assume that, for $\tau = \emptyset$, $\bigcap_{i \in \tau} R_i = R_M$. So in this special case we get that the P_τ modality coincides with the knowledge operator K. Validity and satisfiability is defined analogously as before.

It is possible to axiomatise (the theory of) Mod(**S5P**) as follows (cf. [MH91a, MH93a]): take the **S5** system for the modality K (and dual M) and use K45 for the P-modalities (both the P_i and the P_τ), together with relating axioms, resulting in the system:

5.2. DEFINITION (system **S5P**). In the following, i ranges over I, and τ over subsets of I. Moreover, $\Box$ is a variable over $\{K, P_i, P_\tau \mid i \in I, \tau \subseteq I\}$; $\heartsuit$ and $\spadesuit$ range over $\{i, \tau \mid i \in I\ \tau \subseteq I\}$.

(B1) All propositional tautologies;

(B2) $(\Box\varphi \wedge \Box(\varphi \rightarrow \psi)) \rightarrow \Box\psi$

(B3) $\Box\varphi \rightarrow \Box\Box\varphi$

(B4) $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$;

(B5) $K\varphi \rightarrow \varphi$;

(B6) $KP_\heartsuit\varphi \leftrightarrow P_\heartsuit\varphi$

(B7) $\neg P_\heartsuit\bot \rightarrow (P_\heartsuit P_\spadesuit\varphi \leftrightarrow P_\spadesuit\varphi)$;

(B8) $P_i\varphi \leftrightarrow P_{\{i\}}\varphi$

(B9) $P_\tau\varphi \rightarrow P_{\tau'}\varphi$ $\quad\quad \tau \subseteq \tau'$

(B10) $P_\emptyset \leftrightarrow K\varphi$.

(R1) Modus Ponens

(R2) Necessitation for K: $\vdash \varphi \Rightarrow \vdash K\varphi$.

Axiom (B1) says that we are dealing with an extension of propositional logic; (B2) says that all the operators K, P_i, and P_τ are 'normal'; (B3 and B4) express that the relations R, R_i and R_τ are transitive and Euclidean, respectively; (B5) says that R is reflexive; (B6) and (B7) help us to get rid of nested modalities: a nested modality is always referring to the frame corresponding to the innermost one. (B8) provides us with a bridge between the modalities with indices from I and those of P(I); it also shows that in fact we could do without the P_i's. (B9) says that if $\tau \subseteq \tau'$, then $R_{\tau'} \subseteq R_\tau$. Finally, (B10) is the syntactical counterpart of our definition that $R_\emptyset = R_M$.

We call the resulting system **S5P**. In the sequel we will write $\Gamma \vdash_{\mathbf{S5P}} \varphi$ or $\varphi \in \mathrm{Th}_{\mathbf{S5P}}(\Gamma)$ to indicate that φ is an **S5P**-consequence of Γ. We mean this in the more liberal sense: it is allowed to derive φ from the assertions in Γ by means of the axioms and rules of the system **S5P**, *including the necessitation rule.* (So, in effect we consider the assertions in Γ as additional axioms: $\Gamma \vdash_{\mathbf{S5P}} \varphi$ iff $\vdash_{\mathbf{S5P} \cup \Gamma} \varphi$.)

As for S5, one can again prove a normal form theorem for **S5P** implying that all formulas can be represented equivalently as formulas without nestings.

5.3. THEOREM $\Gamma \vdash_{\mathbf{S5P}} \varphi \Leftrightarrow$ (for all $\mathbb{M} \in \mathrm{Mod}(\mathbf{S5P})$: $\mathbb{M} \models \Gamma \Rightarrow \mathbb{M} \models \varphi$)

PROOF. Combine the arguments given in [MH92a, MH93a] concerning the K- and P_i-modalities with the observations about subrelations given in [HM92].

6. Epistemic Default Logic (EDL)

In the language of **S5P** we express defaults of the form $\varphi : \psi / \chi$ (using Reiter's notation, Cf. [Rei80]) as $\varphi \wedge M\psi \rightarrow P_i\chi$, for some $i \in I$. Here φ, ψ and χ are objective formulas. The reading of such a formula is "if φ is true and ψ is (considered) possible, then χ is preferred (within frame M_i)". Usually we consider cases where ψ is syntactically equal to χ (the so-called *normal* defaults).

By combining the formal apparatus of S5P with Halpern & Moses' nonmonotonic epistemic entailment we obtain a framework in which we can perform default reasoning. In this paper we call this framework Epistemic Default Logic (**EDL**).

6.1. DEFINITION (default theory). A default theory Θ is a pair (W, Δ), where W is a finite, consistent set of objective (i.e. non-modal) formulas describing (necessary) facts about the world, and Δ is a finite set of defaults of the form $\varphi \wedge M\psi \rightarrow P_i\chi$, where φ, ψ and χ are again objective formulas, and $i \in I$. The sets W and Δ are to be considered as sets of axioms, and we may apply necessitation to the formulas in them.

In principle, it would be possible to use the index i of the P-operator in a completely arbitrary way, to be chosen by the knowledge engineer. However, in order to be able to treat the various defaults in a default theory separately from each other, in the sequel we shall assume all P-operators in a default theory to be distinct. This will allow for a generic way to look at the possibilities of combining default beliefs by using the P_τ-operators. So, for example rather than to look at a default theory with $\Delta = \{p \wedge Mq \rightarrow P_1q, r \wedge Ms \rightarrow P_1s\}$, in which for both defaults the same operator P_1 is used, we consider the set $\Delta' = \{p \wedge Mq \rightarrow P_1q, r \wedge Ms \rightarrow$

P_2s}, in which both defaults are represented by means of different operators P_1 and P_2, and consider the combined operator $P_{\{1,2\}}$.

6.2. DEFINITION (default entailment). Let a default theory $\Theta = (W, \Delta)$ be given. Let W* be the conjunction of the formulas in W. Note that W is a finite set, and moreover that W only consists of objective formulas. Furthermore, let φ be an objective formula. Then we define the *default entailment relation* $\vdash\!\sim_\Theta$ w.r.t. default theory Θ as follows: (Recall that $\kappa(\varphi)$ stands for the $\vdash\!\sim$-closure of φ, where $\vdash\!\sim$ is the nonmonotonic epistemic entailment of section 4.)

$$\varphi \vdash\!\sim_\Theta \psi \Leftrightarrow_{def} \kappa(\varphi \wedge W^*) \cup \Delta \vdash_{\mathbf{S5P}} \psi.$$

This definition states that, given a default theory Θ, ψ is a default consequence of φ iff ψ follows in the S5P-logic from the defaults together with what is implied by knowing only the conjunction of φ with the background information W.

Notice that the following equivalence holds, which states something about a modularity of the entailment relations.

6.3 PROPOSITION

$\varphi \vdash\!\sim_\Theta \psi \quad \Leftrightarrow \quad$ there exists a **S5**-formula $\psi' \in$ FORM such that $\varphi \wedge W^* \vdash\!\sim \psi'$ and $\{\psi'\} \cup \Delta \vdash_{\mathbf{S5P}} \psi$

PROOF. This follows directly from the definition of $\vdash\!\sim_\Theta$ and Prop. 4.5. ∎

Instead of **true** $\vdash\!\sim_\Theta \psi$, we simply write $\vdash\!\sim_\Theta \psi$. Note that, for $\Theta = (W, \Delta)$ we have $\varphi \vdash\!\sim_\Theta \psi$ iff $\vdash\!\sim_{\Theta'} \psi$, with $\Theta' = (W \cup \{\varphi\}, \Delta)$. Furthermore, if Γ is a finite set of epistemic formulas, and ψ an S5P-formula, then we define $\Gamma \vdash\!\sim_\Theta \psi$ as $\Gamma^* \vdash\!\sim_\Theta \psi$, where Γ^* stands for the conjunction of the formulas in Γ.

6.4. EXAMPLE (Tweety). Consider the following default theory $\Theta = (W, \Delta)$ with $W = \{p \rightarrow \neg f\}$ and $\Delta = \{b \wedge Mf \rightarrow Pf\}$, representing that penguins do not fly, and that by default birds fly. (For convenience we omit the subscript of the P-operator.) Now consider the following inferences):

(i). $b \vdash\!\sim b \wedge \neg K \neg f \vdash_{\mathbf{S5P}} b \wedge Mf \vdash_{\mathbf{S5P}} Pf$, i.e., $b \vdash\!\sim_\Theta Pf$, meaning that from the mere fact that Tweety is a bird, we conclude that Tweety is assumed to fly. (Here $\mid\!\sim$ stands for Halpern & Moses' nonmonotonic epistemic entailment.) This must be contrasted to the inference:

(ii). $b \wedge p \mid\!\sim Kp \vdash_{S5P} K\neg f \vdash_{S5P} \neg Mf \nvdash_{S5P} Pf$, i.e, **not** $b \wedge p \mid\!\sim_{\Theta} Pf$,

meaning that in case Tweety is a penguin, we cannot infer that Tweety is assumed to fly, but instead we can derive that we know for certain that Tweety does not fly: $b \wedge p \mid\!\sim_{\Theta} K\neg f$.

7. A temporal formalization of downward reflection

In the previous section it has been described how upward reflection can be formalized by a nonmonotonic inference based on epistemic states, and the meta-level process by a (monotonic) epistemic logic. In the current section we will introduce a formalization of the downward reflection step in the reasoning pattern. The meta-level reasoning can be viewed as the part of the reasoning pattern where it is determined what the possibilities are for default assumptions to be made, based on which information is available and (especially) which is not. The outcome at the meta-level concerns conclusions of the form Pφ, where φ is an object-level formula. What is still missing is the step where the default assumptions are actually made, i.e., where such formulas φ are added to the object level knowledge, in order to be able to reason further with them at the object level. Here we actually "jump (down) to conclusions". This is what should be achieved by the downward reflection step. Thus the reasoning pattern as a whole consists of a process of generating possible default assumptions and actually assuming them.

By these downward reflections at the object level a hypothetical world description is created and refined. This means that in principle not all knowledge available at the object level can be derived from the object level theory W: downward reflection creates an essential extension to the object level theory. Therefore it is excluded to model downward reflection according to reflection rules as sometimes can be found in the literature, e.g., "If at the meta-level it is provable that Provable(φ) then at the object level φ is provable" (e.g., see [Wey80]):

$$\frac{MT \vdash \text{Provable}(\varphi)}{OT \vdash \varphi}$$

A reflection rule like this can only be used in a correct manner if the meta-theory about provability gives a sincere axiomatisation of the object level proof system, and in that case by downward reflection nothing can be added to the object level that was not already derivable from the object level theory. In the above rule the meta-theory is only concerned with one fixed object level theory. Since we essentially extend the object level theory, and consequently want to move to another object level theory, an approach as taken in the rule above cannot serve our purposes here.

In fact, in our case a line of reasoning at the object level is modelled by inferences from *subsequently chosen theories* instead of inferences from one fixed theory, where in the choice of the next object-level theory the upward and downward reflections play a role. Another difference with the above "traditional" approach to meta-level reasoning is that we shall treat these reflection steps model-theoretically instead of proof-theoretically, as shifts from one model to another, which results in a *temporal* "super"-model indicating how these shifts may take place over time. Actually, to keep in perfect line with the above, we should explain by means of these models how object knowledge (modelled by a set M of valuations) is reflected upwards to get a model for epistemic meta-knowledge (modelled by an S5-model $\Phi(M)$), how this meta-knowledge is then extended with meta-knowledge about default beliefs (modelled by S5P-models), and how, subsequently, this meta-knowledge is reflected downwards to object knowledge (modelled by a new set M' of valuations) again. However, representing such shifts in a temporal model would involve (sequences of) three different models (sets of valuations, S5-models and S5P-models), which would be quite involved and cumbersome. Therefore, in order not to complicate our temporal models too much, in our present setting we shall not represent the above three steps separately, but treat them "in one blow", so to speak, i.e., as one "super"-step. Now we are able to represent these "super"-steps as shifts from S5P-models to S5P-models, so that our temporal "super"-model only involves sequences of one kind of model, viz. S5P-models, of which the S5-reducts represent the reasoner's objective knowledge and epistemic meta-knowledge, and the frames ('P-') parts represent meta-knowledge in the form of default (or preferred) beliefs. We shall still refer to these supersteps as *downward reflection*, although as we stated above, apart from a reflection of the "meta-knowledge" in the form of default beliefs contained in the S5P-model at hand to the object level by converting some of these beliefs into object knowledge, it also includes an immediate upward reflection from the next object level theory (or rather model) resulting from this to the next meta-level theory as represented by an S5P-model again.

In [GTG93] such a shift between theories is formalized by using an explicit parameter referring to the specific theory (called 'context' in their terms) that is concerned, and by specifying relations between theories. In their case reflection rules ('bridge rules' in their terms) may have the form:

$$\frac{MT \vdash \text{Provable}(OT', \varphi)}{OT' \vdash \varphi}$$

Here it is assumed that at the meta-level, knowledge is available to derive conclusions about provability relations concerning a variety of object level theories OT. So, if at the object level from a (current) theory OT some conclusions have been derived, and these conclusions have been transformed to the meta-level, then the meta-level may derive conclusions about provability from another object level theory OT'. Subsequently one can continue the object level reasoning from this new object level theory OT'. The shift from OT to OT' is introduced by use of the above reflection rule.

As we said above, in the approach as adopted here we give a *temporal* interpretation to these shifts between theories. This can be accomplished by formalizing downward reflection by *temporal logic* (as in [Tre94]). In a simplified case, where no branching is taken into account, the following temporal axioms can be used to formalize downward reflection:

$$P\varphi \rightarrow X\varphi$$

for every objective formula φ. Here X is the temporal operator asserting that in the next (epistemic) state its argument is true.

In the general case we want to take into account branching and the role to be played by an index τ in $P_\tau\varphi$. We will use this index τ to label branches in the set of time points. By combining EDL with the temporal logic obtained in this manner we obtain a formalization of the whole reasoning pattern.

We start (following [FG92]) by defining the temporalized models associated to any class of models and apply it to the classes of models as previously discussed.

In contrast to the reference as mentioned we use labeled flows of time. We use one fixed set L of labels, viz. $L = 2^I$, the powerset of the index set I. However, in most definitions we do not use this fact, but only refer to (elements τ of) L.

7.1 Flows of time

7.1. DEFINITION (discrete labeled flow of time).
Suppose L is a set of labels. A *(discrete) labeled flow of time* (or *lft*) , labeled by L is a pair $\mathbb{T} = (T, (<_\tau)_{\tau \in L})$ consisting of a nonempty (countable) set T of time points and a collection of binary relations $<_\tau$ on T. Here for s, t in T and τ in L the expression $s <_\tau t$ denotes that t is a (immediate) *successor* of s with respect to an arc labeled by τ. Sometimes it is convenient to leave the indices out of consideration and use just the binary relation $s < t$ denoting that $s <_\tau t$

for some τ (for some label τ they are connected). Thus we have that $< = \cup_\tau <_\tau$. We also use the (nonreflexive) transitive closure « of this binary relation: $« = <^+$.

We will make additional assumptions on the flow of time; for instance that it describes a discrete tree structure, with one root and in which time branches in the direction of the future.

7.2. DEFINITION (labeled time tree)
An lft $\mathbb{T} = (T, (<_\tau)_{\tau \in L})$ is called a *labeled time tree* (*ltt*) if the following additional conditions are satisfied (recall that $< = \cup_\tau <_\tau$):

(i) the graph $(T, <)$ is a directed rooted tree.

(ii) *(successor existence)*

Every time point has at least one successor:
for all $s \in T$ there exists a τ and a $t \in T$ such that $s <_\tau t$

(iii) *(label-deterministic)*

For every label τ there is at most one τ-successor:
for all s, t, t' $\in$ T it holds: $s <_\tau t,\ s <_\tau t' \Rightarrow t = t'$

There are still some additional properties that sometimes are required.

7.3. DEFINITION (sub-lft and (maximal) branch)

a) An lft $(T', (<'_\tau)_{\tau \in L})$ is called a *sub-lft* of an lft $(T, (<_\tau)_{\tau \in L})$ if $T' \subseteq T$ and for all τ it holds $<'_\tau = <_\tau \cap T' \times T'$. It is also called the sub-lft of $(T, (<_\tau)_{\tau \in L})$ *defined by* T', or the *restriction of* $(T, (<_\tau)_{\tau \in L})$ to T'.

b) A *branch* in an lft $\mathbb{T}$ is a sub-lft $\mathbb{B} = (T', (<'_\tau)_{\tau \in L})$ of T such that:

(i) $«' = « \cap T' \times T'$ is a total ordering on $T' \times T'$

(ii) Every t' in T' with a <-successor in T also has a <-successor in T':
for all $t' \in T', t \in T : t' < t \Rightarrow$ there is a $s' \in T' : t' < s'$

(iii) Every element of T that is in between elements of T' is itself in T':
for all $s' \in T', t \in T, u' \in T' : s' « t « u' \Rightarrow t \in T'$

c) A branch is called *maximal* if every t' in T' with a τ-predecessor in T also has a τ'-predecessor in T' for some τ': for all $t \in T, t' \in T' : t <_\tau t'$, there is a τ' and an $s' \in T'$: $s' <_{\tau'} t'$. (Note that if T is a labeled time tree, this condition can be simplified to: the root r of T is also the root of T'.)

We now immediately have the following:

7.4. PROPOSITION Any branch of an ltt $\mathbb{T}$ is an ltt.

7.5. DEFINITION

a) An lft is called *successor-complete* if for every label τ every time point has at least one τ-successor: for all s and τ there exists a t such that $s <_\tau t$.

b) A *path* is a finite sequence of successors: $s_0, \ldots, s_n$ such that: $s_i < s_{i+1}$ for all $0 \le i \le n-1$. We call s_0 the starting point and s_n the end point of the path.

7.6. DEFINITION (standard ltt)

The *standard ltt* $\mathbb{S}$ over L is the set of all finite sequences over L equipped with the successor relations:

$$(\tau_0, \tau_1, \ldots, \tau_k) <_\tau (\tau_0, \tau_1, \ldots, \tau_k, \tau),$$

and the empty sequence () as root.

7.7. DEFINITION (embedding and isomorphism). Let $\mathbb{T}$ and $\mathbb{T}'$ be two ltt's. A mapping $f : \mathbb{T}' \to \mathbb{T}$ is an *embedding* if it is injective and successor-preserving: $s <_\tau t$ iff $f(s) <_\tau f(t)$. An embedding is an *isomorphism* if it is surjective.

7.8. PROPOSITION

Every ltt $\mathbb{T}$ is uniquely embeddable in $\mathbb{S}$ (mapping $\mathbb{T}$'s root to the root () of $\mathbb{S}$). Moreover, every successor-complete ltt is isomorphic to $\mathbb{S}$.

This proposition implies that every element t in an ltt is uniquely described by the sequence of labels of a (unique) path from the root to t.

7.9. PROPOSITION

In an ltt for every time point t the intersection of all maximal branches containing t is the set $\{s \mid s \ll t\} \cup \{t\}$.

7.10. DEFINITION (time stamps). Given an ltt $(T, (<_\tau)_{\tau \in L})$, a mapping $|\cdot| : T \to \mathbb{N}$ is called a *time stamp mapping* if for the root r it holds that $|r| = 0$, and for all time points s, t it holds $s < t \Rightarrow |t| = |s| + 1$. (Note that this time stamp mapping is unique.)

Note that an ltt is *infinitely deep*, i.e., for every $k \in \mathbb{N}$ there is a time point $t \in T$ with $|t| = k$. This is a direct consequence of the following proposition:

7.11. PROPOSITION

If $\mathbb{B}$ is a maximal branch in an ltt, then any time stamp mapping is an isomorphism between $\mathbb{B}$ and $\mathbb{N}$.

7.2 Temporal models

We first define our temporal formulas:

7.12. DEFINITION (temporal formulas).

a) Given a logic **L**, *temporal formulas over* (the language of) **L** are defined as follows:

i if φ is a formula of **L** then $C\varphi$ is a temporal formula (also called a *temporal atom*)

ii if φ and ψ are temporal formulas, then so are:

$\neg\varphi$, $\varphi \wedge \psi$, $\varphi \to \psi$, $X_{\exists,\tau}\varphi$, $X_{\exists}\varphi$, $X_{\forall,\tau}\varphi$, $X_{\forall}\varphi$, $F_{\exists}\varphi$, $F_{\forall}\varphi$, $G_{\exists}\varphi$, $G_{\forall}\varphi$.

b) Below, our main concern will be temporal formulas over **S5P**; we will also refer to them as TEDL-formulas (temporal epistemic logic formulas).

The above temporal operators are fairly standard in branching-time temporal logic: X refers to next-time, F to sometimes in the future and G to always in the future; the subscripts refer to whether one considers only some ($\exists$) possible path (possibly with a fixed label τ) or all ($\forall$) of these. The C-operator is the least standard: it means 'currently'. Usually in temporal logic this operator is not really useful, since it just states that its argument, say φ, holds in the current state, which is normally represented by the formula φ itself, without a C in front. Here, however, we have a logic where we mix temporal and epistemic logic, and the C-operator acts as a kind of separator between the epistemic and temporal part: in $C\varphi$, its argument φ is an epistemic formula, while $C\varphi$ itself is a temporal formula. This also facilitates the semantic definition below.

7.13. DEFINITION (temporal models)

a) Let MOD be a class of models, and $\mathbb{T} = (T, (<_\tau)_{\tau \in L})$ a labeled flow of time.
A *temporal* MOD-*model over* $\mathbb{T}$ is a mapping $\mathbb{M}$: T $\to$ MOD. If $\mathbb{M}$ is a temporal MOD-model for any class MOD we call $\mathbb{M}$ a *temporal model*.
For $t \in T$ we sometimes denote $\mathbb{M}(t)$ (the snapshot at time point t) by $\mathbb{M}_t$. The temporal model can alternatively be denoted by $(\mathbb{M}_t)_{t \in T}$.

b) If we apply a) to the classes of models ModSet(**PC**), Mod(**S5**) and Mod(**S5P**), we call these temporalized models *temporal valuation-set-models* (abbreviated *temporal V-models*), *temporal* S5-*models* and *temporal* S5P-*models over* $\mathbb{T}$, respectively.

c) Given an lft $\mathbb{T}$, the temporal formulas are interpreted on MOD-models over $\mathbb{T}$ as follows:

i conjunction and implication are defined as expected; moreover

$\mathbb{M}, s \models \neg\varphi$ iff not $\mathbb{M}, s \models \varphi$;

ii The temporal operators are interpreted as follows:

1) $C\varphi$ means that in the current state φ is true, i.e.

$\mathbb{M}, s \models C\varphi$ iff $\mathbb{M}_s \models \varphi$

2) $X_{\exists,\tau}\varphi$ means that φ is true in some τ-successor state i.e.,

$\mathbb{M}, s \models X_{\exists,\tau}\varphi$ iff there exists a time point t with $s <_\tau t$ such that $\mathbb{M}, t \models \varphi$

3) $X_\exists\varphi$ means that there is a τ with some τ-successor in which φ is true.
$\mathbb{M}, s \models X_\exists\varphi$ iff there exists a time point t with $s < t$ such that $\mathbb{M}, t \models \varphi$

4) $X_{\forall,\tau}\varphi$, meaning that φ is true in all τ-successor states, i.e.,
$\mathbb{M}, s \models X_{\forall,\tau}\varphi$ iff for all time points t with $s <_\tau t$ it holds $\mathbb{M}, t \models \varphi$

5) $X_\forall\varphi$ means that φ is true in all immediate successors:
$\mathbb{M}, s \models X_\forall\varphi$ iff for all time points t with $s < t$ it holds $\mathbb{M}, t \models \varphi$

6) $F_\exists\varphi$ means that φ is true in some future state, i.e.,
$\mathbb{M}, s \models F_\exists\varphi$ iff there exists a time point t with $s \ll t$ such that $\mathbb{M}, t \models \varphi$

7) $F_\forall\varphi$, means that for all future paths there is a time point where φ is true, i.e.,
$\mathbb{M}, s \models F_\forall\varphi$ iff for all branches $\mathbb{B}$ starting in s there is a t in $\mathbb{B}$ with $\mathbb{M}, t \models \varphi$.

8) $G_\exists\varphi$ means that φ is true along some future path, i.e.,
$\mathbb{M}, s \models G_\exists\varphi$ iff there exists a branch $\mathbb{B}$ starting in s with $\mathbb{M}, t \models \varphi$ for all t in $\mathbb{B}$.

9) $G_\forall\varphi$, means that φ is true all future states i.e.
$\mathbb{M}, s \models G_\forall\varphi$ iff for all time points t with $s \ll t$ it holds $\mathbb{M}, t \models \varphi$.

Note that the operator C enforces a shift in the evaluation of formulas; taking us from a temporal model $\mathbb{M}$ and a time point t to an S5P-model $\mathbb{M}_t$.

During the reasoning process we assume to gradually extend the information we have at the object level, and consequently to shrink the set of possible worlds by means of reflecting default beliefs downwards to object knowledge. In terms of temporal S5-models we can formulate this property as follows:

7.14. DEFINITION. A temporal S5P-model *obeys downward reflection*, if the following holds for any s and τ :

the frame M_τ in $\mathbb{M}_s$ is non-empty $\Leftrightarrow$ there is a t with $s <_\tau t$ and for all such t the set of worlds of $\mathbb{M}_t$ equals M_τ

The above property expresses that the possible worlds in frame M_τ (representing the P_τ-default beliefs) are taken to be the whole set of possible worlds (representing the objective knowledge) in some (τ-) successor epistemic state (or, rather, S5P-model).

Finally, we are ready to zoom in into the models we like to consider here, the temporal epistemic default logic models.

7.15. DEFINITION (TEDL-models)

A TEDL-model $\mathbb{M}$ is a temporal S5P-model over an lft $\mathbb{T}$ such that:

1) $\mathbb{T}$ is a labeled time tree;

2) For every time point s, there is exactly one t with $s <_{\varnothing} t$;
3) $\mathbb{M}$ obeys downward reflection.

The following notion is a crucial one for the sequel:

7.16. DEFINITION (conservativity and limit models). Let $\mathbb{T} = (T, (<_\tau)_{\tau \in L})$ be a labeled flow of time. A temporal V-model $\mathbb{M}$ over $\mathbb{T}$ is *conservative* if for every two time points s and t with $s < t$ it holds that $\mathbb{M}(s) \supseteq \mathbb{M}(t)$.
Suppose $\mathbb{M}$ is a conservative temporal V-model. The intersection of the models $\mathbb{M}(s)$ for all s in a given (maximal) branch $\mathbb{B} = (T', (<'_\tau)_{\tau \in L})$ of the lft $\mathbb{T}$ is called the *limit model* of the branch, denoted $\lim_{\mathbb{B}} \mathbb{M}$. The set of limit models for all (maximal) branches is called the *set of limit models* of M. These definitions straightforwardly extend to S5- and S5P-models, by identifying $\mathbb{M}$ with its set of worlds, M.

7.17. THEOREM. TEDL-models are conservative.
PROOF. Suppose $s < t$, that is, for some label τ, $s <_\tau t$. Since TEDL-models obey downward reflection and the underlying lft is an ltt, this means that *the* (now unique) set of worlds M_t of $\mathbb{M}_t$ is the intersection frame M_τ of $\mathbb{M}_s$. Since $M_\tau \subseteq M_s$, the set of worlds of $\mathbb{M}_s$, this gives $\mathbb{M}_t \subseteq \mathbb{M}_s$, which proves the theorem. ■

Since we know that TEDL-models are conservative, we have that once we have obtained that somewhere in a time point s along some path of such a model some objective formula is known, it remains known in all successor points of s. (This is in fact the rational behind the name "conservative models".) We can make this more precise if we introduce so-called *cko-formulas*, which are formulas of the form $CK\varphi$, where φ is objective. In the sequel we will denote cko-formulas by α. Then we have that:

7.18. PROPOSITION. Let $\mathbb{M}$ be a TEDL-model, and α a cko-formula.
If, for some s, $\mathbb{M}, s \models \alpha$, then $\mathbb{M}, t \models \alpha$ for all t with $s \ll t$.
PROOF. Suppose $\mathbb{M}, s \models \alpha$, for $\alpha = CK\varphi$. We show that for t with $s < t$ we have that $\mathbb{M}, t \models \alpha$. (Then we can finish the proof by induction on the number of time point between s and t.) By Theorem 7.17, we have that $\mathbb{M}_t \subseteq \mathbb{M}_s$. This yields: $\mathbb{M}, s \models \alpha \Leftrightarrow \mathbb{M}, s \models CK\varphi \Leftrightarrow \mathbb{M}_s \models K\varphi \Leftrightarrow \mathbb{M}_s, m \models \varphi$ for all m in $\mathbb{M}_s \Rightarrow \mathbb{M}_t, m \models \varphi$ for all m in $\mathbb{M}_t \Leftrightarrow \mathbb{M}_t \models K\varphi \Leftrightarrow \mathbb{M}, t \models CK\varphi \Leftrightarrow \mathbb{M}, s \models \alpha$. ■

Furthermore, TEDL-models enjoy a number of properties that can be expressed as validities in our logic:

7.19. THEOREM. TEDL-models satisfy the following validities:

T0 All the operators of $\{X_{\forall,\tau}, X_\forall, F_\forall, G_\forall\}$ satisfy the K-axiom (C too) and generalisation;

T1 $\vdash_{S5P} \varphi \Rightarrow \vdash_{TEDL} C\varphi$ (introduction of C)

T2 $\neg X_\forall \bot$ (successor existence)

T3 $X_{\exists,\tau}\varphi \rightarrow X_{\forall,\tau}\varphi$ (label-deterministic)

T3' $X_{\exists,\varnothing}\varphi \leftrightarrow X_{\forall,\varnothing}\varphi$ ($\varnothing$-successor existence & label-deterministic)

T4 $X_{\forall,\tau}\varphi \leftrightarrow \neg X_{\exists,\tau}\neg\varphi$ (duality)

T5 $X_\forall\varphi \leftrightarrow \neg X_\exists\neg\varphi$ (duality)

T6 $X_\forall\varphi \leftrightarrow \bigwedge_{\tau\subseteq I} X_{\forall,\tau}\varphi$ ($<$ is union of $<_\tau$)

T7 $X_\exists\varphi \leftrightarrow \bigvee_{\tau\subseteq I} X_{\exists,\tau}\varphi$ (dual of T6)

T8 $C(\neg P_\tau\bot \wedge P_\tau\varphi) \leftrightarrow X_{\exists,\tau}CK\varphi$, if φ is objective (allowing downward reflection)

T8' $CK\varphi \leftrightarrow X_{\exists,\varnothing}CK\varphi$, if φ is objective (trivial downward reflection)

T9 $(C\varphi \rightarrow X_\forall C\varphi) \wedge (CK\varphi \rightarrow X_\forall CK\varphi)$, if φ is objective (conservativity)

T10 $G_\forall\varphi \rightarrow X_\forall\varphi$ ($< \subseteq \ll$)

T11 $G_\forall\varphi \rightarrow X_\forall G_\forall\varphi$ (since $\ll$ is transitive closure of $<$)

T12 $G_\forall(\varphi \rightarrow X_\forall\varphi) \rightarrow (X_\forall\varphi \rightarrow G_\forall\varphi)$ (induction)

T13 $CK\varphi \rightarrow G_\forall CK\varphi$ (from conservativity and induction)

7.20. REMARK. The Theorem above says that the formulas T1 - T13 are at least *sound*; until now we have not been concerned by designing a logic that is complete for TEDL-models.

8. TEDL models of default theories and entailment relations

We could formulate definitions of entailment of objective formulae related to any model, or any model based on the standard tree. But it may well happen that there are branches in such models, for instance labeled by the empty set only, that contain no additional information as compared to the background knowledge. It is not realistic to base entailment on such informationally poor branches in a model. Therefore we define:

8.1. DEFINITION (informationally maximal)
We define for TEDL-models $\mathbb{M}_1$ and $\mathbb{M}_2$ over the same flow of time that $\mathbb{M}_2$ is *informationally larger* than $\mathbb{M}_1$, $\mathbb{M}_1 \leq \mathbb{M}_2$, if for all t it holds $\mathbb{M}_2(t) \subseteq \mathbb{M}_1(t)$.
We call $\mathbb{M}_1$ *informationally maximal* in a class $\mathbf{M}$ of models if it is itself the only model that is informationally larger.

For a given model $\mathbb{M}$ we will apply this definition to the set $\mathbf{B}$ of all maximal branches, with $\mathbb{T} = (T, (<_\tau)_{\tau \in L})$ as flow of time.

8.2. DEFINITION (regular model)

A TEDL-model $\mathbb{M}$ is called *regular* if all maximal branches are informationally maximal in the class of maximal branches. The submodel based on all time points t included in at least one maximal, informationally maximal branch is called the *regular core* of $\mathbb{M}$, denoted by reg($\mathbb{M}$).

8.3. REMARK.
In general the regular core will not be label-complete, because branches may be cut off. The idea behind taking informationally complete branches is that we want to maximalise the effect of applying defaults in order to obtain as much (default) knowledge as possible.

8.4. DEFINITION.
Let $\mathbb{M}$ be a TEDL-model over $\mathbb{T} = (T, (<_\tau)_{\tau \in L})$.
We define for $k \in \mathbb{N}$

$$\mathbb{M}^{(k)} = \bigcup_{t \in \text{reg}(\mathbb{M}),\, |t| = k} \mathbb{M}'(t),$$

$$\mathbb{M}^{\omega} = \bigcap_{k \in \mathbb{N}} \mathbb{M}^{(k)},$$

where $\mathbb{M}'(t)$ stands for the S5-reduct of the S5P-model $\mathbb{M}(t)$.

8.5. PROPOSITION.
Let $\mathbb{M}$ be a TEDL-model.
a) For $k \leq k'$ it holds $\mathbb{M}^{(k')} \subseteq \mathbb{M}^{(k)}$
b) $\mathbb{M}^{\omega} = \bigcup_{\mathbb{B} \text{ maximal branch of reg}(\mathbb{M})} \lim_{\mathbb{B}} \mathbb{M}$
PROOF. a) follows directly from Theorem 7.17, b) follows from the distributivity of intersection over union (and vice versa). ■

8.6. DEFINITION (sceptical entailment)
Let $\mathbb{M}$ be a TEDL-model and α a cko-formula. We define the *sceptical entailment relation* by:
$\mathbb{M} \models_{scep} \alpha \Leftrightarrow$ for every maximal branch $\mathbb{B}$ in reg($\mathbb{M}$) there is a t in $\mathbb{B}$ such that $\mathbb{M}, t \models \alpha$.

An immediate consequence of this definition is:

8.7. PROPOSITION.
If, for some $k \in \mathbb{N}$, $\mathbb{M}^{(k)} \models K\varphi$, then $\mathbb{M} \models_{scep} CK\varphi$.
PROOF. Suppose $\mathbb{M}^{(k)} \models K\varphi$, for some $k \in \mathbb{N}$. This means that $\mathbb{M}'(t) \models K\varphi$ for all S5-reducts $\mathbb{M}'(t)$ with t in reg($\mathbb{M}$) and $|t| = k$. So for every maximal branch $\mathbb{B}$ in reg($\mathbb{M}$) we can find a t in $\mathbb{B}$ such that $\mathbb{M}, t \models K\varphi$ (viz. take t in $\mathbb{B}$ such that $|t| = k$. By Prop. 7.11 we can always do this). Hence, $\mathbb{M} \models_{scep} CK\varphi$. ■

8.8. PROPOSITION.
Let $\mathbb{M}$ be a TEDL-model with root r. Let $\alpha = CK\varphi$ be a cko-formula.

The following are equivalent:

(i) $\mathbb{M} \models_{scep} \alpha$

(ii) reg($\mathbb{M}$), r $\models F_\forall\alpha$

(iii) $\mathbb{M}^\omega \models K\varphi$

(iv) $\lim_\mathbb{B} \mathbb{M} \models K\varphi$ for every maximal branch $\mathbb{B}$ of the regular core of $\mathbb{M}$.

PROOF. Clearly (i) and (ii) are equivalent.By Prop. 8.5 also (iii) and (iv) are equivalent. We now show the equivalence of (i) and (iv). First we prove (i) ⇒ (iv): Suppose $\mathbb{M} \models_{scep} CK\varphi$, i.e. for every maximal branch $\mathbb{B}$ in reg($\mathbb{M}$) there is a t in $\mathbb{B}$ such that $\mathbb{M}$, t $\models CK\varphi$. Now consider a maximal branch $\mathbb{B}$ in reg($\mathbb{M}$). Then there is a t in $\mathbb{B}$ with $\mathbb{M}$, t $\models CK\varphi$, i.e. $\mathbb{M}(t) \models K\varphi$. By the conservativity of $\mathbb{M}$ we have that $\mathbb{M}(u) \models K\varphi$ for all u » t, i.e., all worlds of the models $\mathbb{M}(u)$ with u » t satisfy φ. But then also all worlds of the models $\mathbb{M}(u)$ with u » t and u in $\mathbb{B}$ satisfy φ. Consequently, $\lim_\mathbb{B} \mathbb{M} = \bigcap_{u \text{ in } \mathbb{B}} \mathbb{M}(u) \models K\varphi$.

Next we prove (iv) ⇒ (i): Suppose $\lim_\mathbb{B} \mathbb{M} \models K\varphi$ for every maximal branch $\mathbb{B}$ of the regular core of $\mathbb{M}$. We have to prove that for every maximal branch $\mathbb{B}$ in reg($\mathbb{M}$) there is a t in $\mathbb{B}$ such that $\mathbb{M}$, t $\models CK\varphi$. Take some maximal branch $\mathbb{B}$ in reg($\mathbb{M}$). Then we know that $\lim_\mathbb{B} \mathbb{M} \models K\varphi$, i.e., $\bigcap_{u \text{ in } \mathbb{B}} \mathbb{M}(u) \models K\varphi$. By the conservativity of the model $\mathbb{M}$ (Theorem 7.17) we know that the sequence $\langle \mathbb{M}(u) \mid u \text{ in } \mathbb{B} \rangle$ is monotonically decreasing (with respect to $\subseteq$). Let $\mathbf{P}_0 \subseteq \mathbf{P}$ the set of propositional atoms occurring in the formula α (or φ). Clearly, $\mathbf{P}_0$ is finite. Now, for any S5-model $\mathbb{M}$ with set S of worlds (truth assignment functions) we let $\mathbb{M}_0$ denote the model with set S_0 of worlds: $S_0 = \{t \mid t = s \mid_{\mathbf{P}_0} \text{ for some } s \in S\}$. Clearly, $\mathbb{M} \models K\varphi$ iff $\mathbb{M}_0 \models K\varphi$. Now consider the sequence $\langle \mathbb{M}(u)_0 \mid u \text{ in } \mathbb{B} \rangle$. This is a monotonically decreasing sequence with intersection $\bigcap_{u \text{ in } \mathbb{B}} \mathbb{M}(u)_0$. Since $\mathbf{P}_0$ is finite, and we identified worlds with truth assignment functions, we have that all the models $\mathbb{M}(u)_0$ contain only a *finite* number of worlds. Together with the fact that the sequence is monotonically decreasing this yields that the sequence must be stable from some point s in $\mathbb{B}$ on: $\mathbb{M}(u)_0 = \mathbb{M}(s)_0$ for all u » s. But then obviously the intersection $\bigcap_{u \text{ in } \mathbb{B}} \mathbb{M}(u)_0 = \mathbb{M}(s)_0$. So now $\mathbb{M}(s)_0 \models K\varphi$, and thus $\mathbb{M}(s) \models K\varphi$, i.e. $\mathbb{M}$, s $\models CK\varphi$. ■

For our definition of credulous entailment we can be less restrictive. Especially, too less information in one branch can always be overcome by another, informationally larger branch.

8.9. DEFINITION (credulous entailment)

Let $\mathbb{M}$ be a TEDL-model. We define:

$\mathbb{M} \models^k_{cred} CK\varphi$ ⇔ there exists an s with $|s| = k$ and $\mathbb{M}(s) \models K\varphi$

$\mathbb{M} \models_{cred} CK\varphi$ ⇔ there exists an s with $\mathbb{M}(s) \models K\varphi$

8.10. PROPOSITION.

Let $\mathbb{M}$ be a TEDL-model, and $\alpha = CK\varphi$ a cko-formula.

The following are equivalent:
(i) $\mathbb{M} \models_{cred} \alpha$
(ii) $\mathbb{M} \models^k_{cred} \alpha$ for some $k \in \mathbb{N}$.
(iii) $\mathbb{M}, r \models F_{\exists}\alpha$
(iv) $\lim_{\mathbb{B}} \mathbb{M} \models K\varphi$ for some maximal branch $\mathbb{B}$

PROOF. Since clearly (i), (ii) and (iii) are equivalent, we concentrate on the equivalence of (ii) and (iv): we first prove (ii) $\Rightarrow$ (iv): Let $\mathbb{M} \models^k_{cred} CK\varphi$ for some $k \in \mathbb{N}$. This means that there exists an s with $|s| = k$ and $\mathbb{M}(s) \models K\varphi$, i.e., $\mathbb{M}, s \models CK\varphi$. By Prop. 7.18, we then have that $\mathbb{M}, t \models CK\varphi$ for all t with $s \ll t$. Now consider any maximal branch $\mathbb{B}$ through s. Clearly, this branch has the property that the models $\mathbb{M}(t)$ with $s \ll t$ all satisfy $K\varphi$, i.e., φ holds in every world of every model $\mathbb{M}(t)$. But then φ holds in the intersection of these models as well. This proves $\lim_{\mathbb{B}} \mathbb{M} \models K\varphi$.
Next we prove (iv) $\Rightarrow$ (ii): Suppose that $\lim_{\mathbb{B}} \mathbb{M} \models K\varphi$ for some maximal branch $\mathbb{B}$. This means that $K\varphi$ holds in the intersection of the models $\mathbb{M}(t)$ along $\mathbb{B}$. By the conservativity of the model $\mathbb{M}$ (Theorem 7.17) we know that the sequence $<\mathbb{M}(t) \mid t \text{ in } \mathbb{B}>$ is monotonically decreasing (with respect to $\subseteq$). Let $\mathbf{P}_0 \subseteq \mathbf{P}$ the set of propositional atoms occurring in the formula α (or φ). Clearly, $\mathbf{P}_0$ is finite. As above, we denote, for any S5-model $\mathbb{M}$ with set S of worlds (truth assignment functions), the model with set S_0 of worlds by $\mathbb{M}_0$ where $S_0 = \{t \mid t = s \mid_{\mathbf{P}_0} \text{ for some } s \in S\}$. Again, $\mathbb{M} \models K\varphi$ iff $\mathbb{M}_0 \models K\varphi$. Now consider the sequence $<\mathbb{M}(u)_0 \mid u \text{ in } \mathbb{B}>$. This is a monotonically decreasing sequence with intersection $\bigcap_{u \text{ in } \mathbb{B}} \mathbb{M}(u)_0$. Since $\mathbf{P}_0$ is finite, and we identified worlds with truth assignment functions, we have that all the models $\mathbb{M}(t)_0$ contain only a finite number of worlds. Together with the fact that the sequence is monotonically decreasing this yields that the sequence must be stable from some point s on: $\mathbb{M}(t)_0 = \mathbb{M}(s)_0$ for all $t \gg s$. But then obviously the intersection $\bigcap_{t \text{ in } \mathbb{B}} \mathbb{M}(t)_0 = \mathbb{M}(s)_0$. Since the intersection satisfies $K\varphi$, we obtain that $\mathbb{M}(s)_0 \models K\varphi$, and thus $\mathbb{M}(s) \models K\varphi$. Hence $\mathbb{M} \models_{cred} CK\varphi$. ■

We can now associate TEDL-models with default theories as follows.

8.11. DEFINITION (TEDL-model of a default theory).
Let $\Theta = (W, \Delta)$ be a default theory. Then we define a TEDL-model of Θ as a TEDL-model $\mathbb{M}^\Theta$ such that:
(i) (basis: the root) $\mathbb{M}^\Theta_r$ is an S5P-model such that (a) the S5-reduct of $\mathbb{M}^\Theta_r$ is the S5-model $\mathbb{M}_{W*}$, as defined in Section 4, and (b) $\mathbb{M}^\Theta_r$ satisfies the set of defaults, i.e., $\mathbb{M}^\Theta_r \models \Delta$.
(ii) (induction step) Suppose that we are given an S5P-model at snapshot $\mathbb{M}^\Theta_s$. Then we have that for a(n S5P-) model $\mathbb{M}^\Theta_t$ with $s <_\tau t$, it holds that: (a) the S5-reduct of $\mathbb{M}^\Theta_t$ is the S5-

model $\mathbb{M}_\tau$ as it appeared as a frame in $\underline{\mathbb{M}}^{\Theta}{}_s$, and (b) $\underline{\mathbb{M}}^{\Theta}{}_t$ satisfies the set of defaults again, i.e., $\underline{\mathbb{M}}^{\Theta}{}_t \models \Delta$.

Note that, in general, there are multiple TEDL-models of a default theory Θ. Furthermore, note that clause (ii)(a) reflects the downward reflection operation with respect to the P_τ-defaults.

This definition enables us to finally give the definitions of sceptical and credulous entailment from a default theory.

8.12. DEFINITION (entailment from a default theory).
Let $\Theta = (W, \Delta)$ be a default theory, and φ an objective formula. Then:

$\Theta \models_{scep} \varphi$ iff for all models $\underline{\mathbb{M}}$ of Θ it holds $\underline{\mathbb{M}} \models_{scep} CK\varphi$

$\Theta \models_{cred} \varphi$ iff for all models $\underline{\mathbb{M}}$ of Θ it holds $\underline{\mathbb{M}} \models_{cred} CK\varphi$

Of course, we have that what is entailed sceptically, also is entailed credulously:

8.13. PROPOSITION. For default theory Θ and objective formula φ: $\Theta \models_{scep} \varphi \Rightarrow \Theta \models_{cred} \varphi$

9. Conclusions

In [MH93a,b] an Epistemic Default Logic (**EDL**) was introduced inspired by the notion of meta-level architecture that also was the basis for the BMS-approach introduced in [TT91]. In EDL drawing a default conclusion has no other semantics than that of adding a modal formula to the meta-level. No downward reflection takes place to be able to reason with the default conclusions at the object level (by means of which default assumptions actually can be made). In [TT91] downward reflection takes place, but no logical formalization was given: it was defined only in a procedural manner.

In principle downward reflection disturbs the object level semantics, since facts are added that are not logically entailed by the available knowledge. Adding a temporal dimension (in the spirit of [FG92]) to **EDL** enables one to obtain formal semantics of drawing a default conclusion in a dynamic sense: as a transition from the current object level theory to a next one (where the default conclusion has been added). This view, also underlying the work presented in [ET93, ET94] and [Tre94], turns out to be very fruitful. It turns out that a number of notions can be formalized in temporal semantics in a quite intuitive and transparent manner. As an example in the current paper we formalized the notions of sceptical and credulous entailment on the basis of temporal models.

Acknowledgements

Discussions about the subject with Joeri Engelfriet have played a stimulating role for the development of the material presented here.

REFERENCES

[Doh91] P. Doherty, NM3 - A Three-Valued Cumulative Non-Monotonic Formalism, in: *Logics in AI*, (J. van Eijck, ed.), LNCS 478, Springer, Berlin, 1991, pp. 196-211.

[ET93] J. Engelfriet, J. Treur, A Temporal Model Theory for Default Logic, in: M. Clarke, R. Kruse, S. Moral (eds.), *Proc. 2nd European Conference on Symbolic and Quantitative Approaches to Reasoning and Uncertainty, ECSQARU '93*, Springer Verlag, 1993, pp. 91-96; Extended version: Report IR-321, Vrije Universiteit Amsterdam, Department of Mathematics and Computer Science, 1993.

[ET94] J. Engelfriet, J. Treur, Temporal Models of Reasoning, in: *Proc. JELIA'94,* LNCS 838, Springer-Verlag, 1994, pp. 277-299..

[FG92] M. Finger, D. Gabbay, Adding a temporal dimension to a logic system, *Journal of Logic, Language and Information* 1 (1992), pp. 203-233.

[GTG93] E. Giunchiglia, P. Traverso, F. Giunchiglia, Multi-Context Systems as a Specification Framework for Complex Reasoning Systems, in: J. Treur, Th. Wetter (eds.), *Formal Specification of Complex Reasoning Systems*, Ellis Horwood, 1993, pp. 45-72

[HM84] J.Y. Halpern & Y.O. Moses, Towards a Theory of Knowledge and Ignorance, *Proc. Workshop on Non-Monotonic Reasoning*, AAAI, 1984b, pp. 125-143.

[HMT94] W. van der Hoek, J.-J. Ch. Meyer & J. Treur, Formal Semantics of Temporal Epistemic Reflection, *Proc. META'94*, Pisa, LNCS 883, Springer Verlag, 1994.

[MH91a] J.-J.Ch. Meyer & W. van der Hoek, Non-Monotonic Reasoning by Monotonic Means, in: J. van Eijck (ed.), *Logics in AI (Proc. JELIA '90)*, LNCS 478, Springer, 1991, pp. 399-411.

[MH92a] J.-J.Ch. Meyer & W. van der Hoek, A Modal Logic for Nonmonotonic Reasoning, in: W. van der Hoek, J.-J. Ch. Meyer, Y.H. Tan & C. Witteveen (eds.), *Non-Monotonic Reasoning and Partial Semantics*, Ellis Horwood, Chichester, 1992, pp. 37-77.

[MH93a] J.-J. Ch. Meyer & W. van der Hoek, An Epistemic Logic for Defeasible Reasoning using a Meta-Level Architecture Metaphor. Techn. Report IR-329, Free University, Amsterdam, 1993.

[MH93b] J.-J. Ch. Meyer & W. van der Hoek, A Default Logic Based on Epistemic States. in: M. Clarke, R. Kruse, S. Moral (eds.), *Proc. 2nd European Conference on Symbolic and Quantitative Approaches to Reasoning and Uncertainty, ECSQARU '93*, Springer Verlag, 1993, pp. 265-273.

[MH94] J.-J. Ch. Meyer & W. van der Hoek, *Epistemic Logic for AI and Computer Science*, forthcoming.

[Moo85] R.C. Moore, Semantical Considerations on Nonmonotonic Logic, *Artificial Intelligence* 25, 1985, pp. 75-94.

[Rei80] R. Reiter, A Logic for Default Reasoning, *Artificial Intelligence* 13, 1980, p.81-132.

[Rei87] R Reiter, Nonmonotonic Reasoning, *Annual Reviews of Computer Science* 2, 1987, pp. 147-187.

[Sho87] Y. Shoham, A Semantical Approach to Nonmonotonic Logics, in *Proceedings 10th IJCAI*, 1987, pp. 388-392.

[Sho88] Y. Shoham, *Reasoning about Change*, MIT Press, Cambridge, Massachusetts, 1988.

[TT91] Y.-H. Tan & J. Treur, A Bi-Modular Approach to Non-Monotonic Reasoning, in: *Proc. WOCFAI'91* (M. DeGlas & D. Gabbay, eds.), Paris, 1991, pp. 461-475.

[Tre94] J. Treur, Temporal Semantics of Meta-level Architectures for the Control of Reasoning, *Proc. META'94*, Pisa, LNCS 883, Springer Verlag, 1994.

[Wey80] R.W. Weyhrauch, Prolegomena to a Theory of Mechanized Formal Reasoning, *Artificial Intelligence J.* 13 (1980), pp. 133-170