Selection Schemes, Elitist Recombination, and Selection Intensity

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Abstract

Selection algorithms used in evolutionary computation can be characterized according to two features: pure versus elitist selection schemes, and generational versus steady-state selection schemes. Recently the concept of selection intensity has been shown to be a convenient quantitative measure of the selection pressure of pure generational reproduction methods. Here we will discuss how this measure can also be used for elitist and steady-state selection mechanisms. A second goal of the paper is to generalise the Elitist Recombination genetic algorithm such that its selective pressure can be tuned, and to compute the selection intensity of the proposed method. Finally we conclude by computing the selection intensity of a reproductive scheme where both fitness biased parent selection and fitness biased replacement is used.

1 INTRODUCTION

All search algorithms can be characterized by the way they handle the so-called exploitation-exploration tradeoff. Exploitation refers to the tendency of the algorithm to steer its search direction by the information it has obtained in previous steps, while exploration indicates how and how much search will be dedicated to new, unexplored parts of the search space. In evolutionary search algorithms the exploitation of accumulated information is done by the selection mechanism, while the exploration of new regions of the search space is accounted for by the variation operators, namely crossover and mutation. The tradeoff between exploitation and exploration is mainly determined by the selective pressure or the relative proportion of individuals that are selected from the pool of individuals created by the variation operators. This selective pressure can conveniently be quantified by computing the *selection intensity* of the selection mechanism.

The exploitation-exploration balance is also influenced by the way the selection phase and the variation phase are connected. In the standard genetic algorithm the two phases are completely independent of each other: selection picks out the better individuals and variation creates new offspring from them. There is no competition between the parents and the children so the offspring replaces the parents irrespective of their fitness values, and all individuals have a life time of exactly one generation. In contrast to this *pure* selection scheme there is also the *elitist* selection scheme where the offspring have to compete with their parents to gain admission to the new population. One advantage of the elitist scheme is that good solutions once found are never lost unless even better solutions are created. In the pure selection scheme the parents are always thrown away, so there is a distinct possibility that valuable information is lost.

This paper is organized as follows. In the next section we review the selection intensity concept and discuss its use in elitist and steady-state genetic algorithms. We also give a short review of the theory of order statistics. Section 3 presents the generalised version of the Elitist Recombination GA, and computes its selection intensity when implemented as a generational and as a steady-state algorithm. Section 4 shows the computation of the selection intensity for a replacement based strategy.

2 BACKGROUND

2.1 SELECTION INTENSITY

A convenient way to quantify the selection pressure of a selection scheme is by computing its *selection intensity I*. This concept originates from the field of quantitative genetics (Bulmer, 1985; Falconer, 1989), and has been introduced into the evolutionary computation community by Mühlenbein and Schlierkamp-Voosen (1993) who computed the selection intensity for truncation (or (μ, λ)) selection. The selection intensity for tournament selection (with tournament size $s = 2$) was calculated by Thierens and Goldberg (1994b), and important generalizations were made by Bäck (1995) and by Miller and Goldberg (1995) using the theory of order statistics. Blickle and Thiele (1995) derived the selection intensity for linear and exponential ranking.

Quantitative genetics studies the inheritance of those differences between individuals that are quantitative rather than qualitative. Quantitative differences have a continuous nature such as the height or the weight of the human body, whereas qualitative variation is measured in discrete units or categories such as eye color or blood type. To characterize the evolution of the quantitative differences the following concepts are defined. The selection progress or response to selection $R(t)$ is defined as the difference between the mean fitness of the population at generation $t+1$ and the population mean fitness at generation t. The selection differential $S(t)$ is the difference between the mean fitness of the parent population at generation t and the population mean fitness at generation t. Assuming that the population fitness is normally distributed $N(\overline{f}, \sigma^2)$ we can scale the selection differential by the standard deviation $\sigma(t)$. This scaled selection differential is called the selection intensity $I(t)$ and is dimensionless since the standard deviation has the units in which the selection response is expressed:

$$I(t) = \frac{S(t)}{\sigma(t)} = \frac{\overline{f^s}(t) - \overline{f}(t)}{\sigma(t)}.$$

Standardizing the normal distribution ($\overline{f} = 0, \sigma = 1$) shows that the selection intensity I is simply the expected average fitness of the population after applying the selection scheme to a population with standardized normal distributed fitness ($N(0,1)$). The relation between the response to selection R and the selection differential S is given by the heritability h^2:

$$R(t) = h^2 S(t),$$

or

$$R(t) = h^2 \sigma(t) I(t).$$

For generational selection schemes the selection intensity I can immediately be used to quantify the selective pressure. However for steady-state and elitist selection schemes we need to be more precise:

1. In a steady-state GA the offspring of two selected parents are immediately placed into the population. Strictly speaking we cannot apply the selection intensity concept because there is no population of selected parents where we can calculate the population mean fitness increase. To make comparisons with the generational schemes meaningful we define the selection intensity I for steady-state selection schemes as the expected average fitness of the population (of size n) after n children have been generated (this is after $n/2$ matings) starting from a population with standardized normal distributed fitness ($N(0,1)$).

2. In elitist selection schemes the offspring competes with the parents to be included in the next generation (or in the current population for an elitist steady-state algorithm). Because of this intertwining of parents and children we can only measure the response to selection R and not the selection differential S or the selection intensity I. If however the heritability factor h^2 is 1 then R and S are equal and the selection intensity I can now easily be computed and measured. A straightforward way to make the heritability factor h^2 equal to 1 is simply to take a copy of the parents as children.

With these additional assumptions we can calculate the selection intensity for steady-state and elitist selection schemes. In the next section we will first give a short review of the theory of order statistics.

2.2 ORDER STATISTICS

The theory of order statistics describes the statistical properties of a set of random variables that are ordered according to their value. Since most selection schemes are based on the relative order of the individuals according to their fitness value, order statistics is a very useful analytical tool in evolutionary computation (Bäck, 1995a, 1995b; Miller & Goldberg, 1995).

Assume we take a random sample of size n of a population with a certain distribution probability, and we sort the sample in increasing order of magnitude:

$$x_{1:n} \leq x_{2:n} \leq \ldots \leq x_{n-1:n} \leq x_{n:n}$$

The *ith* order statistic is the random variable $X_{i:n}$ that represents the distribution of the corresponding value

$x_{i:n}$. The probability density function $p_{i:n}(x)$ of the ith order statistic $X_{i:n}$ gives the probability that the ith ranked individual of a sample of size n will have a value equal to x. Call $P(x)$ the cumulative distribution function of x. Then the probability density function $p_{i:n}(x)$ is given by:

$$p_{i:n}(x) = n \binom{n-1}{i-1} P(x)^{i-1} \; (1 - P(x))^{n-i} \; p(x)$$

which is the probability that one particular sample has value x ($= p(x)$), times the probability that from the remaining $n-1$ samples exactly $i-1$ have a value lower than or equal to x, or $\binom{n-1}{i-1} \; P(x)^{i-1} \; (1 - P(x))^{n-i}$, times the number of samples n. The expected value $u_{i:n}$ of the ith order statistic $X_{i:n}$ is thus:

$$\begin{aligned} u_{i:n} &= \int_{-\infty}^{+\infty} x \; p_{i:n}(x) \; dx \\ &= n \binom{n-1}{i-1} \int_{-\infty}^{+\infty} x p(x) P(x)^{i-1} (1 - P(x))^{n-i} dx \end{aligned}$$

Since the selection intensity is defined by applying the selection procedure on a population with standardized normal distributed fitness $N(0, 1)$, we need to compute the order statistics for this case. The probability density function $p(x)$ becomes

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

and the cumulative distribution $P(x)$ is

$$\Phi(x) = \int_{-\infty}^{x} \phi(x) dx.$$

The probability density function $p_{i:n}(x)$ of the ith order statistic $X_{i:n}$ is thus given by:

$$p_{i:n}(x) = n \binom{n-1}{i-1} \Phi(x)^{i-1} \; (1 - \Phi(x))^{n-i} \phi(x) \quad (1)$$

while the expected value $u_{i:n}$ of the ith order statistic becomes:

$$u_{i:n} = n \binom{n-1}{i-1} \int_{-\infty}^{+\infty} x \phi(x) \Phi(x)^{i-1} (1 - \Phi(x))^{n-i} dx \quad (2)$$

3 ELITIST RECOMBINATION

3.1 ELITIST RECOMBINATION GA

Elitist Recombination is an evolutionary algorithm where the children compete with their parents to be included in the next population (Thierens & Goldberg, 1994a). There is no separate selection and recombination phase but only a competition in each family, which typically consist of two mating parents and their two offspring[1]. The best two of each family survive and are included in the next population. A number of advantages of elitist recombination can be noted (Thierens & Goldberg, 1994a; van Kemenade et al., 1996). First, good solutions are never lost during the search process. Second, elitist recombination is less sensitive to undersized populations than tournament selection with standard recombination. For a tournament size $s = 2$ the selection intensity of both mechanisms is nevertheless the same. Third, there is no need to choose a particular value for the crossover probability p_c. Offspring can only replace their parents when they are fitter so choosing $p_c = 1$ has no deleterious effect on the growth of building blocks. Fourth, elitist recombination is extremely simple to implement and well suited for parallel GA implementations. Finally, elitist recombination is ideal to be used in hybrid GAs where the genetic algorithm's global search is combined with the local search approach of more classical optimization algorithms to obtain the best of both worlds.

The concept of holding a competition between the parents and their offspring has appeared in a number of papers although quite interestingly the reason for introducing this *family competition* has been different in each case. Mahfoud (1992) proposed an algorithm called deterministic crowding where a parent competes with his most similar child according to a genotypic (or phenotypic) distance measure. This resulted in a crowding algorithm with little stochastic replacement errors. In latter work he also made use of the family competition approach to design a genetic algorithm that incorporated the principles of simulated annealing by holding the parent-child competition according to the Metropolis acceptance criterium (Mahfoud & Goldberg, 1992; Mahfoud & Goldberg, 1995).

Culberson (1993) proposed the Genetic Invariance Genetic Algorithm (GIGA) where the pair of children competes with the parent pair. By replacing both parents by their offspring, the number of alleles in the population remains constant.

Finally Altenberg (1994) suggests upward-mobility selection where a recombinant offspring is only placed in the population when it is fitter than its parents. Altenberg's aim is "to put deleterious and near-neutral

[1]Note that it is straightforward to use Elitist Recombination with multiparent recombination. With q-ary recombination q offspring are created and the family competition is now held between the q parents and q children.

recombinations on an equal footing, thus preventing the proliferation of code that trades-off its evolvability value for an ability not to produce deleterious effects". Upward-mobility selection and Elitist Recombination are basically equivalent implementations of the family elitism idea.

In this paper we would like to generalise the Elitist Recombination GA so that its selective pressure can be tuned. Although one could immediately propose a number of mechanisms to obtain an increased selection pressure, care has to be taken in order to preserve the characteristics of the algorithm:

- First, we need to preserve the family elitism: the children have to compete directly with their parents.
- Second, it should be impossible for individuals to make copies of themselves. Consequently we cannot increase the selective pressure by interchanging regular tournament selection with elitist recombination.
- Third, every individual should have a reasonable chance of becoming selected as parent, because this is the only way that they can be replaced by a better individual. As a result we cannot increase the selection pressure by biasing the parent selection according to their fitness. The higher the selective pressure the less probable it would become that weak individuals are selected, so they only rarely have to compete to stay in the population. This would mean that we have virtually reduced the population size.

In accordance with the above constraints we propose to generalise the Elitist Recombination GA as follows: the first parent is selected according to its fitness by holding a tournament of size s and picking the best individual; the second parent is simply chosen at random. In the next section we will compute the selection intensity of this generalised Elitist Recombination GA.

3.2 SELECTION INTENSITY

The generalised Elitist Recombination algorithm selects one parent at random while the other parent is selected according to its fitness. This biased selection is done by holding a tournament between s randomly chosen individuals. The most fit within the tournament becomes the fitness biased parent and gets mated with the randomly chosen parent. The winners of the family competition go to the next generation when using a generational scheme, or they are placed in the current population when using a steady-state scheme. First we will compute the selection intensity for the generational scheme.

3.2.1 Generational Elitist Recombination

To compute the selection intensity we need to calculate the expected fitness value of the parents. The best fit parent is the one with the highest fitness of the s individuals in the tournament plus the randomly selected parent. Since the tournament members are also randomly selected, the expected value of the fittest parent is the $(s+1)^{th}$ order statistic of the random sample of size $s+1$, or $\mu_{s+1:s+1}$. Recalling our assumption that the two children are just copies of the two parents it is obvious that the two best of the family are the fittest parent and its copy.

The generational Elitist Recombination scheme places these two individuals into the next generation and then repeats the process until the entire next population is complete. The expected population mean fitness increase is thus $\mu_{s+1:s+1}$ which is also the selection intensity I because the standard deviation of our starting population is 1:

$$I = \mu_{s+1:s+1}$$

In table 1 we have shown the selection intensity I for different tournament sizes s.

Table 1: Selection intensity I for different tournament sizes s of generational Elitist Recombination

s	1	2	3	4	5
I	0.56	0.85	1.03	1.16	1.27

3.2.2 Steady-state Elitist Recombination

Contrary to the generational method, the steady-state Elitist Recombination scheme replaces the winners of the family competition immediately back into the population. The two parents are thus replaced by the two winning individuals. As in the generational scheme the two winners will be the fittest parent and its child copy, so the algorithm reduces to replacing the least fit parent by a copy of the best parent. This replacement will increase the population mean fitness with $(\mu_h - \mu_l)/n$ where μ_h (μ_l) is the expected fitness value of the best (least) fit parent and n the population size.

Recall that the selection intensity for steady-state schemes was defined as the population mean fitness increase after n children have been generated, so the selection intensity of the steady-state Elitist Recombination algorithm becomes:

$$I = \frac{\mu_h - \mu_l}{2}$$

As explained in the above section the expected value of the best parent is simply:

$$\mu_h = \mu_{s+1:s+1}.$$

To compute the expected value of the least fit parent we need to calculate the probability density function of the least fit parent:

$$p_l(x) = \phi(x)(1 - P_{s:s}(x)) + p_{s:s}(x)(1 - \Phi(x)).$$

The first term computes the probability that the least fit parent is the randomly selected parent. The probability that the randomly selected parent has a fitness value x is $\phi(x)$ since the population has a standard normalized fitness distribution. The probability that the fitness biased parent - this is the winner of the tournament - has a fitness value higher than x is $1 - P_{s:s}(x)$, with $P_{s:s}(x)$ the cumulative distribution function of the s_{th} order statistic of a random sample of size s. The probability density function of the s_{th} order statistic is

$$p_{s:s}(x) = s\Phi(x)^{s-1}\phi(x)$$

so the cumulative distribution $P_{s:s}(x)$ becomes

$$P_{s:s}(x) = \int_{-\infty}^{x} p_{s:s}(x)dx.$$

The second term in the probability density function of the least fit parent expresses the probability that the least fit parent is the fitness biased parent or the winner of the tournament. The probability that the winner of the tournament has a fitness value x is $p_{s:s}(x)$, and the probability that the randomly selected parent has a fitness value higher than x is $1 - \Phi(x)$, with $\Phi(x)$ the cumulative distribution function of standard normal distribution. Integrating the probability density function $p_l(x)$ gives us the expected value of the least fit parent:

$$\mu_l = \int_{-\infty}^{+\infty} x \; p_l(x) \; dx.$$

In table 2 we compute the selection intensity values for a number of tournament sizes by numerically integrating the above equations.

Table 3 compares the computed selection intensities with their experimental value. The experimental values are obtained from measuring the fitness increase on the bit-counting (or one-max) problem with a string length $l = 50$ and a population size $n = 500$. The selection intensity I is then:

$$\begin{aligned} I^{exp} &= \frac{\Delta f(t)}{\sigma(t)} \\ &= \frac{\Delta f(t)}{\sqrt{lp(t)(1-p(t))}} \end{aligned}$$

with $p(t)$ the proportion of 1-bits. For the generational case the fitness increase Δf is measured between the random initialized population and the first generation. In the steady-state case the fitness increase Δf is taken after n offspring are generated. Results are averaged over 50 runs.

Table 2: Selection intensity I for different tournament sizes s of Steady-state Elitist Recombination

s	μ_l	μ_h	I
1	-0.56	0.56	0.56
2	-0.28	0.85	0.56
3	-0.18	1.03	0.61
4	-0.13	1.16	0.65
5	-0.10	1.27	0.69

Table 3: Selection intensity I and the experimental value for generational and steady-state Elitist Recombination

s	I_{gen}	I_{gen}^{exp}	I_{steady}	I_{steady}^{exp}
1	0.56	0.57	0.56	0.56
2	0.85	0.85	0.56	0.56
3	1.03	1.02	0.61	0.59
4	1.16	1.16	0.65	0.65
5	1.27	1.27	0.69	0.68

4 SELECTION BY REPLACEMENT

The selection pressure is typically tuned by a fitness biased selection of the parents. An alternative is to tune the selective pressure by a fitness biased selection of the individuals that should be replaced. Examples of GAs with a replacement strategy are Genitor (Whitley,1989), Syswerda's GA (Syswerda, 1991), Eshelman's CHC (Eshelman, 1991) and $(\mu + \lambda)$ (Bäck, 1991). The first two are steady-state implementations while the latter two are generational.

To illustrate the modeling of selection intensity for replacement strategies we consider a rather general and easy to tune selection scheme that combines the fitness biased parent selection with the fitness biased replacement. The algorithm chooses s individuals at random from the population, selects the best two individuals from this set as parents, generates two children, and let them compete with the two worst of the tournament.

Applying the order statistics formula from section 2 it becomes straightforward to compute the selection intensity. First we calculate the probability density function of the best, second best, worst, and second worst individuals in a sample of size s:

$$p_{s:s}(x) = s\Phi(x)^{s-1}\phi(x),$$

$$p_{s-1:s}(x) = s(s-1)\Phi(x)^{s-2}(1-\Phi(x))\phi(x),$$

$$p_{1:s}(x) = s(1-\Phi(x))^{s-1}\phi(x),$$

and

$$p_{2:s}(x) = s(s-1)\Phi(x)(1-\Phi(x))^{s-2}\phi(x).$$

while their expected value is given by

$$u_{s:s} = s \int_{-\infty}^{+\infty} x\ \Phi(x)^{s-1}\ \phi(x)dx,$$

$$u_{s-1:s} = s(s-1) \int_{-\infty}^{+\infty} x\ \Phi(x)^{s-2}\ (1-\Phi(x))\ \phi(x)dx,$$

$$u_{1:s} = s \int_{-\infty}^{+\infty} x\ (1-\Phi(x))^{s-1}\ \phi(x)dx,$$

and

$$u_{2:s} = s(s-1) \int_{-\infty}^{+\infty} x\ \Phi(x)\ (1-\Phi(x))^{s-2}\ \phi(x)dx.$$

Note that since $1-\Phi(x) = \Phi(-x)$ and $\phi(x) = \phi(-x)$ we have

$$u_{i:s} = -u_{s-i+1:s}$$

or

$$\begin{cases} u_{1:s} = -u_{s:s} \\ u_{2:s} = -u_{s-1:s}\ . \end{cases}$$

To calculate the selection intensity we have to distinguish three cases depending on the size of the tournaments:

1. For tournament sizes $s \geq 4$ the best two individuals replace the worst two, so the selection intensity I is given by

$$\begin{aligned} I &= \frac{1}{2}\left((u_{s:s} + u_{s-1:s}) - (u_{1:s} + u_{2:s})\right) \\ &= u_{s:s} + u_{s-1:s} \end{aligned}$$

2. When the tournament size $s = 3$ the second best individual is also the second worst so the expected average fitness increase is now

$$\begin{aligned} I &= \frac{1}{2}\left(u_{3:3} - u_{1:3}\right) \\ &= u_{3:3} \end{aligned}$$

3. Finally for a tournament size $s = 2$ the best individual replaces the second best which gives

$$\begin{aligned} I &= \frac{1}{2}\left(u_{2:2} - u_{1:2}\right) \\ &= u_{2:2} \end{aligned}$$

Table 4 shows the values of I for tournament sizes from 2 to 10. Experimental values were again obtained for the bit-counting function with stringlength $l = 50$ and population size $n = 500$. Results are averaged over 50 runs.

Table 4: Selection intensity and the experimental value for the fitness biased selection and replacement

s	$u_{s:s}$	$u_{s-1:s}$	I	I^{exp}
2	0.56	-	0.56	0.55
3	0.85	-	0.85	0.83
4	1.03	0.29	1.32	1.30
5	1.16	0.50	1.66	1.61
6	1.27	0.64	1.91	1.84
7	1.35	0.76	2.11	2.07
8	1.42	0.85	2.27	2.21
9	1.49	0.93	2.42	2.32
10	1.54	1.00	2.54	2.46

5 DISCUSSION

The aim of this paper was to show how the concept of selection intensity can be used for steady-state and elitist evolutionary algorithms. A second goal was to generalise the Elitist Recombination GA such that its selective pressure can be modified.

Computing the selection intensity of different GA implementations is useful to quantify the selective pressure of the proposed algorithm. Previous work showed how the selection intensity can also be used to predict the dynamical behavior or the convergence process of the GA (Mühlenbein and Schlierkamp-Voosen, 1993; Thierens and Goldberg, 1994b; Bäck, 1995; Miller and Goldberg, 1995; Blickle and Thiele, 1995). This dynamical modeling is however limited because it assumes that the heritabilty h^2 is equal to one (as in the one-max problem), so the selection differential S is equal to the response to selection R.

In future work we plan to compare the heritability of Elitist Recombination with that of tournament selection and traditional recombination on more difficult functions than the one-max problem. We believe that it is exactly here that the benefit of family elitism can be shown.

6 CONCLUSION

We discussed the application of the selection intensity measure on elitist and steady-state selection mechanisms. We also proposed a generalization of the Elitist Recombination genetic algorithm and computed its selection intensity. A second illustration of the selection intensity computation was made by quantifying the selective pressure of a reproductive method where both fitness biased parent selection and fitness biased replacement are used. In addition to the existing work on selection intensity for generational selection schemes, we believe that the work in this paper illustrates how the selection intensity of most evolutionary algorithms can be computed.

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