

Building Probabilistic Networks: Where Do the Numbers Come From? — a Guide to the Literature

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1 Introduction

Probabilistic networks are now fairly well established as practical representations of knowledge for reasoning under uncertainty, as demonstrated by an increasing number of successful applications in such domains as (medical) diagnosis and prognosis, planning, vision, information retrieval, and natural language processing. A probabilistic network (also referred to as a belief network, Bayesian network, or, somewhat imprecisely, causal network) consists of a graphical structure, encoding a domain's variables and the qualitative relationships between them, and a quantitative part, encoding probabilities over the variables [Pearl, 1988].

Building a probabilistic network for a domain of application involves three tasks. The first of these is to identify the variables that are of importance, along with their possible values. Once the important domain variables have been identified, the second task is to identify the relationships between the variables discerned and to express these in a graphical structure. The tasks of eliciting the variables and values of importance as well as

the relationships between them from domain experts is comparable, to at least some extent, to knowledge engineering for other artificial-intelligence representations and, although it may require significant effort, is generally considered doable. The last task in building a probabilistic network is to obtain the probabilities that are required for its quantitative part. This task often appears more daunting: “Where do the numbers come from?” is a commonly asked question. The three tasks in building a probabilistic network are, in principle, performed one after the other. Building a network, however, often requires a careful trade-off between the desire for a large and rich model to obtain accurate results on the one hand, and the costs of construction and maintenance and the complexity of probabilistic inference on the other hand. In practice, therefore, building a probabilistic network is a process that iterates over these tasks until a network results that is deemed requisite.

This paper focuses on the task of obtaining the probabilities required, the most daunting task in building probabilistic networks; the paper basically is a guide to the relevant literature. In Section 2, we describe the various sources of probabilistic information that are typically available for the task. In Section 3, we address the question how accurate the numbers obtained should be to arrive at satisfactory behaviour of a probabilistic network. In Section 5, we review various methods and tools that aim at reducing the burden of network quantification.

2 Sources of Probabilistic Information

In most application domains, probabilistic information is available from various sources. The most common are (statistical) data, literature, and human experts. Despite the abundance of information, these sources seldom provide all numbers required for the quantitative part of a probabilistic network. As a consequence, the task of obtaining the numbers for a real-life application is hard and time consuming.

In data-rich application domains, often large data collections are available, retrospectively documenting every-day problem solving. Once the part of the domain to be modelled is well defined and well demarcated, it also is not too hard to prospectively collect data on the variables of interest. These data will usually contain highly valuable information about the relationships between the variables in the domain. If a comprehensive data collection is available, the construction of both the graphical part and the quantitative part of a probabilistic network can be performed automatically. The basic idea of the former is to distill information about the relationships between the variables from the data and exploit it for constructing the network’s graph. There are essentially two approaches to learning the graphical structure from data. The first is based on constraint-based search [Pearl and Verma, 1991, Spirtes *et al.*, 1993] and the second on Bayesian search for graphs with highest posterior probability given the data [Cooper and Herskovits, 1992]. Once the graphical structure has been established, assessing the required probabilities is quite straightforward and amounts to studying subsets of the data that satisfy various conditions.

To allow for automated construction of a meaningful probabilistic network, the data

must have been collected very carefully. Biases that are introduced in the data as a result of the data collection strategies used will usually have an effect on the resulting network [Lucas *et al.*, 1998]. This effect may not be desirable, however, for the purpose for which the network is being developed. Unfortunately, selection biases are not easily detected in a network, once it has been constructed. Also, the variables and associated values that are recorded in the data collection should match the variables and values that are to be modeled in the network, or should at least admit transformation into these variables and values without too much loss of information [Lucas *et al.*, 2000]. The data collection should further comprise enough data to allow for reliable identification of probabilistic relationships among the variables discerned and to provide for reliable probability assessments. In an insufficiently large data collection, the various subsets from which probabilities are estimated, for example, can be empty or too small to allow for meaningful assessments. A common problem typically found in real-life data, especially when it has been retrospectively collected, is the occurrence of missing values. Sometimes a missing value is the result of an error of omission. Quite often, however, a value is not recorded because the variable's measurement did not make sense in practice given the values of other variables. Missing values of the first type are often randomly distributed. Missing values of the second type, on the other hand, generally are not distributed evenly; as a consequence, they are information bearing and need be handled accordingly [Peot and Shachter, 1998]. To use a data collection with missing values for automated construction of a probabilistic network, often values have to be filled in, for example based upon (roughly) estimated prior or posterior probabilities for these values or with the help of domain experts [Cowell, 1999, Heckerman *et al.*, 1995, Ramoni and Sebastiani, 1998]. Automated construction of probabilistic networks from data is an active area of research [Buntine, 1996, Cowell *et al.*, 1999, Jordan, 1998].

Literature often provides abundant probabilistic information. For every medical diagnostic test, for example, its sensitivity and specificity characteristics as well as its typical ranges are reported in medical handbooks or journals. Medical disorders and symptoms as well as the (causal) relationships between them are also discussed in ample detail. Unfortunately, the reported probabilistic information is seldom directly amenable to encoding in a probabilistic network. Medical literature, for example, often reports conditional probabilities of the presence of symptoms given a disorder, but not always the probabilities of these symptoms occurring in the absence of the disorder. Also, conditional probabilities are sometimes given in a direction reverse to the direction required for the network. For example, the statement "70% of the patients with oesophageal cancer are smokers" specifies the probability of a patient being a smoker given that he or she is suffering from oesophageal cancer, while for the network the probability of oesophageal cancer developing in a smoker would be required. Moreover, probabilities for unobservable intermediate disease states are usually lacking altogether. As a consequence, if the reported probabilistic information can be exploited at all, it often requires considerable processing and additional domain knowledge [Korver and Lucas, 1993]. Another commonly found problem that prohibits direct use of probabilistic information from literature pertains to the characteristics of the population from which the information is derived. These characteristics often are not prop-

erly described or deviate seriously from the characteristics of the population for which the probabilistic network is being developed [Druzdzel *et al.*, 1999]. Almanacs, morbidity and mortality tables, and statistical yearbooks generally suffer less from the problems outlined above: these sources tend to contain fairly reliable probabilistic information that can be used whenever the target population is not atypical.

Finally, when there are few or no reliable data available, the knowledge and experience of experts in the domain of application constitute the only remaining source of probabilistic information. The role of domain experts in the construction of the quantitative part of a probabilistic network should not be underestimated. An expert's knowledge and experience can help, not just in assessing the probabilities required, but also in fine tuning probabilities obtained from other sources to the specifics of the domain at hand, and in verifying the numbers within the context of the network. The problems encountered when directly eliciting probabilities from experts, however, are widely known [Kahneman *et al.*, 1982]. An expert's assessments, for example, may reflect various biases and may not be properly calibrated. Acknowledging these problems, in the field of decision analysis various techniques have been developed for the elicitation of well-calibrated probabilities from experts, ranging from the use of probability scales for marking assessments to the use of lotteries [Morgan and Henrion, 1990, Von Winterfeldt and Edwards, 1986]. These techniques tend to be quite time-consuming and can take up to 30 minutes per number, including the typical overhead in interviews with domain experts, for example, of explaining context. They have found widespread use in the construction of decision-analytic models, which traditionally comprise a reasonably small number of variables. Probabilistic networks tend to differ from conventional decision-analytic models by the number of probabilities they require: contemporary networks typically comprise tens or hundreds of variables and hundreds or thousands of probabilities. Given that an expert's time is a scarce and expensive commodity, application of the decision-analytic techniques for probability elicitation rapidly becomes impractical if not impossible for network quantification. For probability elicitation for probabilistic networks, therefore, supplementary techniques are being sought [Druzdzel and Van der Gaag, 1995, Renooij and Wittman, 1999, Van der Gaag *et al.*, 1999].

To conclude our brief discussion of sources of probabilistic information, we would like to note that, although tempting, combining information from different sources in a single probabilistic network can be risky and can in fact lead to incorrect results [Druzdzel and Díez, 2000].

3 The Importance of Accurate Numbers

Although generally various sources of information can be exploited for probability assessment, the numbers obtained are inevitably inaccurate, due to incompleteness of data and partial knowledge of the domain under study. As the numbers are an integral part of a probabilistic network, their inaccuracies will influence the network's output. It is a natural question, therefore, to ask how accurate the numbers should be to arrive at satisfactory be-

haviour of the network. Experience with constructing probabilistic networks for various domains of application has established a consensus that the graphical structure of a network is its most important part, as it reflects the independence and relevance relationships between the variables concerned, being the most robust, qualitative properties of the domain under study [Dawid, 1976, Druzdzel and Suermondt, 1994, Van der Gaag and Meyer, 1998]. Within the context of the graphical structure, however, numerical inaccuracies will influence the network's output.

The extent to which the inaccuracies in its numbers influence the output of a probabilistic network can be studied by investigating the extent to which deviations from the numbers affect the output. To this end, the network can be subjected to a sensitivity analysis and an uncertainty analysis. In general, sensitivity analysis of a mathematical model amounts to investigating the effects of inaccuracies in the model's parameters on its output, by systematically varying the parameters' values [Morgan and Henrion, 1990]. For a probabilistic network, sensitivity analysis amounts to varying the assessments for one or more probabilities in the network's quantitative part simultaneously and investigating the effects on a probability of interest. In an uncertainty analysis, the assessments for all probabilities are varied simultaneously by drawing, for each of them, a value from a pre-specified distribution. Uncertainty analysis serves to reveal the overall reliability of a network's output, yet yields less insight in the effect of separate probabilities than a sensitivity analysis.

Uncertainty analysis of a large real-life probabilistic network for liver and biliary disease has provided evidence that probabilistic networks can be highly insensitive to inaccuracies in the numbers in their quantitative part [Henrion *et al.*, 1996, Pradhan *et al.*, 1996]. There is additional, sometimes anecdotal, evidence that networks that contain crude assessments for their probabilities exhibit reasonable behaviour. From this evidence, numbers may be looked upon as merely convenient order of magnitude approximations of the strengths of influences between variables. However, evidence is building up that probabilistic networks can be sensitive to the inaccuracies in their numbers. Sensitivity analysis of a real-life network for congenital heart disease, for example, has revealed large effects on a probability of interest [Coupé *et al.*, 1999]. We feel that from the limited available evidence no decisive conclusions can be drawn with respect to the effects of inaccuracies in a network's probabilities. At present, it seems likely that these effects will vary from application to application. Sensitivity analysis and uncertainty analysis of probabilistic networks constitute an active field of research, that has yielded efficient computational methods for studying the robustness of a network's output [Castillo *et al.*, 1997, Kjærulff and Van der Gaag, 2000, Laskey, 1995, Van der Gaag and Coupé, 2000]. With these methods, more experimental results of sensitivity and uncertainty analyses of real-life probabilistic networks are likely to become available in the near future.

4 Reducing the Burden

Typical contemporary probabilistic networks comprise tens or hundreds of variables, easily requiring thousands of probabilities. It is the vast number of probabilities required that generally hampers the construction of a network for a real-life application. Often the majority of these probabilities have to be assessed by domain experts. As we have argued before, the conventional decision-analytic techniques for probability elicitation are too much time consuming to be suitable for the task. In fact, any contemporary or future technique that aims at eliciting well-calibrated and unbiased probability assessments from domain experts is likely to suffer from this problem. We feel, therefore, that research efforts aimed at reducing the number of probabilities to be assessed and at procedures and tools for supporting the quantification task currently are of more practical significance.

The number of probabilities required for a probabilistic network depends directly on the network's graphical structure. Roughly speaking, the more densely connected a network's graph, the more numbers it requires for its quantitative part. For each variable exponentially many probabilities have to be provided, their number being exponential in the size of the variable's parental set. There are essentially two approaches to reducing the number of probabilities that have to be assessed for a network. The first is based on changes to the graphical structure and the other on the use of parametric probability distributions. The former approach builds, for example, on the principle of divorcing parents by introducing intermediate variables [Olesen *et al.*, 1989] and on removal of arcs representing weak dependences [Kjærulff, 1994, Van Engelen, 1997]. The use of a parametric probability distribution for a variable is aimed at reducing the number of probabilities that have to be assessed directly, by providing simple rules for the computation of the other probabilities required. Examples of parametric probability distributions currently in use are modeled by the noisy-OR and noisy-AND gates and their generalisations [Díez, 1993, Heckerman and Breese, 1996, Henrion, 1989, Pearl, 1988, Srinivas, 1993]. These models are based on (inter-)causal independence assumptions for a variable and its parents. The number of probabilities to be assessed directly for a variable with such a model is linear rather than exponential in its number of parents; the remaining, exponentially many, probabilities are readily derived from the independence assumptions underlying the model. A noisy-OR gate, for example, for a binary variable with n binary parents requires n rather than 2^n numbers; for $n = 10$, this means a reduction of the number of probabilities to be assessed directly by two orders of magnitude. Changes to the graphical structure of a probabilistic network and the use of parametric distributions are likely to come at the price of accuracy. There currently is little insight in whether or not a fully detailed network with separately specified assessments has a better performance than a network that is carefully reduced using the approaches outlined above. There is no doubt, however, that the reduced network will have required considerably less time on the part of the experts involved. The time thus saved can be exploited for verifying and refining the network.

Building a probabilistic network requires a careful trade-off between the desire for a large and rich model on the one hand and the costs of construction, maintenance, and inference on the other hand. As we have argued before, building a network is a creative and

iterative process. Although obtaining the numbers for a probabilistic network is generally postponed until its graphical structure is considered robust, it is not realistic to assume that the assessment of all numbers required is a one-shot process. Building upon this insight, various research efforts aim at iterative procedures and associated tools to support the daunting quantification task. The procedures build, for example, on the use of sensitivity and value-of-information analyses [Coupé *et al.*, 2000, Philips, 1982]. As these give insight in the level of accuracy that is required for the various probabilities of a network, they help in focusing elicitation efforts. A procedure building upon sensitivity analysis, for example, sets out with the elicitation of crude, probably highly inaccurate, numbers, within a short period of time [Van der Gaag *et al.*, 1999]. Starting with these numbers, a sensitivity analysis of the network is performed. The most influential probabilities are uncovered, which are thereupon refined for example using conventional decision-analytic elicitation techniques. As a side-effect, the analysis can point to uninfluential parts of a network that may be deleted or simplified. Iteratively performing sensitivity analyses and refining probabilities is pursued until satisfactory behaviour of the network is obtained, until the costs of further elicitation outweigh the benefits of higher accuracy, or until higher accuracy can no longer be attained due to lack of knowledge. Given the limited and costly time of experts, attention can thus be focused on the probabilities to which the network's behaviour shows highest sensitivity.

Procedures for network quantification can be supported by graphical tools that provide for interactive elicitation, inspection, and modification of probabilities [Wang and Druzdzel, 2000]. With such tools, probabilities can be elicited through a variety of modalities. Direct elicitation of probabilities, while easiest to implement, is generally the least reliable. Elicitation using graphical means that allow an expert to directly manipulate a pie chart or a bar graph offers more support to the expert and is likely to lead to numbers with higher accuracy. Probabilities can also be related to verbal descriptions such as *very likely* and *improbable* [Renooij and Wittman, 1999]. Although verbal descriptions of probabilities are known to be context sensitive and can describe wide ranges of numerical quantities, the use of both words and numbers in probability elicitation can result in reasonable assessments [Van der Gaag *et al.*, 1999]. Rather than pushing an expert to assess a large number of probabilities, tools for interactive probability elicitation can support non-invasive elicitation by accommodating whatever probabilistic information the expert is willing to provide [Druzdzel and Van der Gaag, 1995]. This information may be quantitative in nature, such as point estimates and probability intervals, but may also be qualitative, such as comparisons and statements of stochastic dominance. Procedures for network quantification can furthermore be supported by tools for automated generation of explanations of reasoning behaviour. Detailed explanation provides for studying the reasoning behaviour of a network which can point to problems with the probabilities in the network's quantitative part.

To conclude, in this brief introduction we have focused attention on the task of obtaining the numbers required for a probabilistic network as this is the main scope of the current issue. However, as we have argued before, the quantification task is not performed in isolation from the rest of the process of building a network. With the increasing number

of applications, a need for knowledge-engineering principles tailored to the construction of probabilistic networks is emerging. With the advance of iterative procedures and associated graphical tools for supporting the overall construction process, the quantification task will be addressed within its proper context which will, hopefully, contribute to reducing its burden.

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