A Thorough Documentation of Obtained Results on Real–Valued Continuous and Combinatorial Multi–Objective Optimization Problems Using Diversity Preserving Mixture–Based Iterated Density Estimation Evolutionary Algorithms

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Abstract

In this paper, we present the results of performing experiments with three different multi–objective evolutionary algorithms (MOEAs) on eight different optimization problems. As such, this paper is only an extension of an earlier publication in which one of the three MOEAs is introduced [1]. Although the experiments and the obtained results have already been reported in the earlier publication, not all information could be reported due to space limitations. In this paper, we present even more information about the experiments that were performed.

1 Outline

The reader of this paper is assumed to be familiar with the most important concepts related multi–objective optimization such as Pareto dominance. If this is not the case, the reader is advised to first read the main publication regarding this benchmark, which contains such additional information [1], since it is not repeated here.

The three MOEAs that were used for testing are the *Strength Pareto Evolutionary Algorithm* (SPEA) [11] by Zitzler and Thiele, the *Non–Dominated Sorting Genetic Algorithm* (NSGA–II) [4] by Deb *et al.* and the recentmost version of the *Multi–objective Mixture–based Iterated Density Estimation Evolutionary Algorithm* ($\mathbb{MIDEA}$) [1] by Bosman and Thierens. We shall refrain from presenting the details of these algorithms in this paper but refer the interested reader to the indicated literature.

This paper is organized as follows. In section 2 we present the optimization problems that we have tested the MOEAs on. In section 3 we discuss the indicators that were used to measure the performance of the MOEAs. In section 4 we describe the setup of the experiments and in Section 5 we tabulate the results. A discussion of these results may be found elsewhere [1].

2 Multi–Objective optimization problems

Our test suite consists of problems with real–valued variables as well as problems with binary variables. In both cases we have used four different optimization problems and have used two different dimensionalities for these problems to obtain a total test suite size of 16 problems. In the following we give a brief description of the problems in our test suite.

Name	Definition	Range
BT_1	**Minimize** $(f_0(\boldsymbol{y}), f_1(\boldsymbol{y}))$ **Where** • $f_0(\boldsymbol{y}) = y_0$ • $f_1(\boldsymbol{y}) = 1 - f_0(\boldsymbol{y}) + 10^7 - \frac{100}{\left(10^{-5}+\sum_{i=1}^{l-1}\lvert\sum_{j=1}^{i} y_i\rvert\right)}$	• $y_0 \in [0,1]$ • $y_i \in [-3,3]$ $(1 \leq i < l)$
ZDT_4	**Minimize** $(f_0(\boldsymbol{y}), f_1(\boldsymbol{y}))$ **Where** • $f_0(\boldsymbol{y}) = y_0$ • $f_1(\boldsymbol{y}) = \gamma\left(1 - \sqrt{\frac{f_0(\boldsymbol{y})}{\gamma}}\right)$ • $\gamma = 1 + 10(l–1) + \sum_{i=1}^{l-1}\left(y_i^2 - 10\cos(4\pi y_i)\right)$	• $y_0 \in [0,1]$ • $y_i \in [-5,5]$ $(1 \leq i < l)$
ZDT_6	**Minimize** $(f_0(\boldsymbol{y}), f_1(\boldsymbol{y}))$ **Where** • $f_0(\boldsymbol{y}) = 1 - e^{-4y_0}\sin^6(6\pi y_0)$ • $f_1(\boldsymbol{y}) = \gamma\left(1 - \left(\frac{f_0(\boldsymbol{y})}{\gamma}\right)^2\right)$ • $\gamma = 1 + 9\left(\sum_{i=1}^{l-1}\frac{y_i}{9}\right)^{0.25}$	• $y_i \in [0,1]$ $(0 \leq i < l)$
CTP_7	**Minimize** $(f_0(\boldsymbol{y}), f_1(\boldsymbol{y}))$ **Where** • $f_0(\boldsymbol{y}) = y_0$ • $f_1(\boldsymbol{y}) = \gamma\left(1 - \frac{f_0(\boldsymbol{y})}{\gamma}\right)$ • $\gamma = 1 + 10(l–1) + \sum_{i=1}^{l-1}\left(y_i^2 - 10\cos(4\pi y_i)\right)$ **Such That** • $\cos(-\frac{5\pi}{100})f_1(\boldsymbol{y}) - \sin(-\frac{5\pi}{100})f_0(\boldsymbol{y}) \geq 40\lvert\sin(5\pi\left[\sin(-\frac{5\pi}{100})f_1(\boldsymbol{y}) + \cos(-\frac{5\pi}{100})f_0(\boldsymbol{y})\right])\rvert^6$	• $y_0 \in [0,1]$ • $y_i \in [-5,5]$ $(1 \leq i < l)$

Figure 1: Real–valued multi–objective optimization test problems.

2.1 Real–valued multi–objective optimization problems

A variety of test problems for real–valued variables has been proposed that may cause different types of problems for multi–objective optimization algorithms [3, 5, 10]. From this set of problems, we have selected three problems that are commonly used to benchmark multi–objective optimization algorithms. The fourth real–valued test problem is a new test problem proposed by ourselves. These problems represent a spectrum of multi–objective problem difficulty as they make it difficult for a multi–objective optimization algorithm to progress towards the global optimal front and to maintain a diverse spread of solutions due to properties such as discontinuous fronts and multi–modality. The problems with real–valued variables that we use in our experiments are all defined for two objectives. An overview of our test problems is given in Figure 1.

2.1.1 ZDT_4

Function ZDT_4 was introduced by Zitzler *et al.* [10]. It is very hard to obtain the optimal front $f_1(\boldsymbol{y}) = 1 - \sqrt{y_0}$ in ZDT_4 since there are many local fronts. Moreover, the number of local fronts increases as we get closer to the Pareto optimal front. The main problem that a MOEA should be able to overcome to optimize this problem is thus strong multi–modality.

2.1.2 ZDT_6

Function ZDT_4 was also introduced by Zitzler *et al.* [10]. The density of solutions in ZDT_6 increases as we move away from the Pareto optimal front. Furthermore, this function has a non–uniform density of solutions *along* the Pareto optimal front as there are more solutions as $f_0(\boldsymbol{y})$ goes up to 1. Therefore, a good diverse spread of solutions along the Pareto front is hard to obtain. The Pareto front for ZDT_6 is given by $f_1(\boldsymbol{y}) = 1 - f_0(\boldsymbol{y})^2$ with $f_0(\boldsymbol{y}) \in [1 - e^{-1/3}; 1]$.

2.1.3 CTP_7

Function CTP_7 was introduced by Deb *et al.* [5]. Its Pareto optimal front differs slightly from that of ZDT_4, but otherwise shares the multi–modal front problem. In addition, this problem has constraints in the objective space, which makes finding a diverse representation of the Pareto front more difficult since the Pareto front is discontinuous and it is hard to obtain an approximation of a front that has a few solutions in each feasible part of that front.

2.1.4 BT_1

Function BT_1 has not been used before in the field multi–objective optimization. It differs from the other three functions in that it has multivariate (linear) interactions between the problem variables. Therefore, more complex factorizations are required to exploit these interactions, whereas all of the other problems are well–suited to be optimized using the univariate factorization. The Pareto optimal front is given by $f_1(\boldsymbol{y}) = 1 - y_0$.

2.2 Binary multi–objective optimization problems

In Figure 2, four binary multi–objective optimization problems are specified. These problems are multi–objective variants of well–known combinatorial optimization problems. The number of objectives for these problems is not restricted to two and is denoted by m.

It is important to note that we have used random instances for the combinatorial optimization problems. In the case of only a single objective, random instances may on average be easy for some combinatorial problems. However, in the case of multiple objectives, finding the Pareto front is usually much more difficult, even if efficient algorithms are available for the single–objective case [6]. Therefore, the instances used in our test suite are not expected to be over–easy. Furthermore, the problems also serve to indicate differences between the different multi–objective algorithmic approaches other than the fact that dependencies between problem variables can be exploited. This relative performance of the algorithms may be well observed using our proposed test–suite. On the other hand, the degree of interaction between the problem variables in randomly generated problem instances may not be too large, which may cause optimization algorithms that regard the problem variables independently of each other to be the most efficient.

2.2.1 Maximum satisfiability

In the maximum satisfiability problem, we are given a propositional formula in conjunctive normal form. The goal is to satisfy as many clauses as possible. The solution string is a truth assignment to the involved literals. These formulas can be represented by a matrix in which row i specifies what literals appear either positive (1) or negative (-1) in clause i. In the multi–objective variant of this problem, we have m of such matrices and only a single solution to satisfy as many clauses as possible in each objective at the same time.

2.2.2 Knapsack

The multi–objective knapsack problem was first used to test MOEAs on by Zitzler and Thiele [11]. We are given m knapsacks with a specified capacity and n items. Each item can have a different weight and profit in every knapsack. Selecting item i in a solution implies placing it in every knapsack. A solution may not cause exceeding the capacity of any knapsack.

Name	Definition
MS (Maximum Satisfiability)	**Maximize** $(f_0(\boldsymbol{x}), f_1(\boldsymbol{x}), \ldots, f_{m-1}(\boldsymbol{x}))$ **Where** • $\forall_{i \in \boldsymbol{\mathcal{M}}} : f_i(\boldsymbol{x}) = \sum_{j=0}^{c_i-1} \text{sgn}\left(\left[\sum_{k=0}^{l-1} (C_i)_{jk} \otimes x_k\right]\right)$ • $\text{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$ • $\begin{array}{c\|cc} \otimes & 0 & 1 \\ \hline -1 & 1 & 0 \end{array}$ $\begin{array}{c\|cc} \otimes & 0 & 1 \\ \hline 0 & 0 & 0 \end{array}$ $\begin{array}{c\|cc} \otimes & 0 & 1 \\ \hline 1 & 0 & 1 \end{array}$
KN (Knapsack)	**Maximize** $(f_0(\boldsymbol{x}), f_1(\boldsymbol{x}), \ldots, f_{m-1}(\boldsymbol{x}))$ **Where** • $\forall_{i \in \boldsymbol{\mathcal{M}}} : f_i(\boldsymbol{x}) = \sum_{j=0}^{l-1} P_{ij} x_j$ **Such That** • $\forall_{i \in \boldsymbol{\mathcal{M}}} : \sum_{j=0}^{l-1} W_{ij} x_j \leq c_i$
SC (Set Covering)	**Minimize** $(f_0(\boldsymbol{x}), f_1(\boldsymbol{x}), \ldots, f_{m-1}(\boldsymbol{x}))$ **Where** • $\forall_{i \in \boldsymbol{\mathcal{M}}} : f_i(\boldsymbol{x}) = \sum_{j=0}^{l-1} C_{ij} x_j$ **Such That** • $\forall_{i \in \boldsymbol{\mathcal{M}}} : \forall_{0 \leq j < r} : \sum_{k=0}^{l-1} (A_i)_{jk} x_k \geq 1$
MST (Minimal Spanning Tree)	**Minimize** $(f_0(\boldsymbol{x}), f_1(\boldsymbol{x}), \ldots, f_{m-1}(\boldsymbol{x}))$ **Where** • $\forall_{i \in \boldsymbol{\mathcal{M}}} : f_i(\boldsymbol{x}) = \sum_{j=0}^{l-1} W_{ij} x_j$ **Such That** • $\forall_{S \subseteq V} : \sum_{x_j \in (S \times (V-S))} x_j \geq 1$ • $\forall_{S \subseteq V} : \sum_{x_j \in (S \times S)} x_j \leq \|S\| - 1$

Figure 2: Binary multi–objective combinatorial optimization test problems.

2.2.3 Set covering

In the set covering problem, we are given l locations at which we can place some service at a specified cost. Furthermore, associated with each location is a set of regions $\subseteq \{0, 1, \ldots r-1\}$ that can be serviced from that location. The goal is to select locations such that *all* regions are serviced against minimal costs. In the multi–objective variant of set covering, m services are placed at a location. Each service however covers its own set of regions when placed at a certain location and has its own cost associated with a certain location. A binary solution indicates at which locations the services are placed.

2.2.4 Minimal spanning tree

In the minimal spanning tree problem we are given an undirected graph (V, E) such that each edge has a certain weight associated with it. We are interested in selecting edges $E_T \subseteq E$ such that (V, E_T) is a spanning tree. The objective is to find a spanning tree such that the weight of all its edges is minimal. In the multi–objective variant of this problem, each edge can have a different weight in each objective.

3 Performance indicators

To compare the MOEAs, we look at their average performance with respect to three different performance indicators. The most important one is the *average front distance* **AFD**. This metric is the average minimal Euclidean distance over all points in a given default front $(\mathcal{F}_D)$ front to another front $\mathcal{F}$. In the case of the real–valued problems, we know the optimal front. The default front in this case consists of a uniformly sampled set of 5000 solutions along the Pareto optimal front. Since we do not know the Pareto optimal front for the binary optimization problems, we use the Pareto front over all results obtained by all algorithms as the default front. Because the distance is computed over all points in the default front instead of over all points in the front found in a certain run, this measure gives us a sense of how well each part of the optimal or best known front is covered on average. It also gives us a sense of distance to the optimal front.

$$\mathbf{AFD}(\mathcal{F}_D, \mathcal{F}) = \frac{1}{|\mathcal{F}_D|} \sum_{\boldsymbol{x} \in \mathcal{F}_D} \min \left\{ \sum_{i=0}^{m-1} (\boldsymbol{x}_i - \boldsymbol{y}_i)^2 \;\middle|\; \boldsymbol{y} \in \mathcal{F} \right\} \tag{1}$$

If two algorithms obtain a somewhat comparable AFD score, it is interesting to look at other properties of the obtained results. A second metric we use is the *front spread* **FS**. This metric gives a notion of the size of the objective space covered by the Pareto front. It is the maximum Euclidean distance inside the m–dimensional hypercube that is obtained by taking the maximum distance among the points in the front in each dimension:

$$\mathbf{FS}(\mathcal{F}) = \sqrt{\sum_{i=0}^{m-1} \max\{(\boldsymbol{x}_i - \boldsymbol{y}_i)^2 \mid (\boldsymbol{x}, \boldsymbol{y}) \in \mathcal{F} \times \mathcal{F}\}} \tag{2}$$

The third and final metric is the *front occupation* **FO**, which is simply the number of points on the front:

$$\mathbf{FO}(\mathcal{F}) = |\mathcal{F}| \tag{3}$$

4 Experiment setup

In this section we outline the settings that we have used to test the three MOEAs.

4.1 Optimization problem dimensionalities

4.1.1 Real–valued multi–objective optimization problems

For the real–valued problems, we tested all algorithms with both $l = 10$ and $l = 100$ problem variables.

4.1.2 Binary multi–objective optimization problems

For the binary problems, we used test instances with $l = 100$ and $l = 1000$. For the maximum satisfiability problem, we generated the test instances by generating 2500 clauses for $l = 100$ and 12500 clauses for $l = 1000$ with a random number of literals between 1 and 5. For the knapsack problem, we generated instances by generating random weights in $[1; 10]$ and random profits in $[1; 10]$. The capacity of a knapsack was set at half of the total weight of all the items, weighted according to that knapsack objective. For set covering, the costs were generated at random in $[1; 10]$. We used 250 regions and 2500 regions to be serviced for $l = 100$ and $l = 1000$ respectively. We varied the problem difficulty through the region–location adjacency relation. This relation was generated by making each location adjacent to 70 and 50 randomly selected regions for $l = 100$ and $l = 1000$ respectively. Finally, for the minimum spanning tree problem, we used full graphs with 105 edges (15 vertices) and 1035 edges (46 vertices). The dimensionality of these problems is therefore not precisely 100 and 1000. The weights of the edges were generated randomly in $[1, 10]$.

4.2 Optimization problem constraints

Problems CTP_7, set covering, knapsack and minimal spanning tree have constraints. To deal with them, we can use a repair mechanism to transform infeasible solutions into feasible solutions. Another approach is based on the notion of constraint–domination introduced by Deb *et al.* [5].

This notion allows to deal with constrained multi–objective problems according to a very general scheme. A solution $\boldsymbol{z}^0$ is said to *constraint–dominate* solution $\boldsymbol{z}^1$ if any of the following is true:

1. Solution $\boldsymbol{z}^0$ is *feasible* and solution $\boldsymbol{z}^1$ is *infeasible*
2. Solutions $\boldsymbol{z}^0$ and $\boldsymbol{z}^1$ are both *infeasible*, but $\boldsymbol{z}^0$ has a smaller overall constraint violation
3. Solutions $\boldsymbol{z}^0$ and $\boldsymbol{z}^1$ are both *feasible* and $\boldsymbol{z}^0 \succ \boldsymbol{z}^1$

In the above definition, the overall constraint violation is the amount by which a constraint is violated, summed over all constraints. We have used this principle for problems CTP_7 and set covering. For the knapsack problem, an elegant repair mechanism was proposed in earlier MOEA research [11]. For the minimal spanning tree problem, the number of constraints grows exponentially with the problem size l. We therefore propose to use repair mechanisms for these latter two problems.

4.2.1 Knapsack repair mechanism

If a solution violates a constraint, the repair mechanism iteratively removes items until all constrains are satisfied. The order in which the items are investigated, is determined by the maximum profit/weight ratio. The items with the lowest profit/weight ratio are removed first.

4.2.2 Minimal spanning tree repair mechanism

First the edges are removed from the currently constructed graph and they are sorted according to their weight. Next, they are added to the graph such that no cycles are introduced. This is done by only allowing edges to be introduced *between* the connected components in the graph. If after this phase, the number of connected components has not been reduced to 1, all edges between the connected components are regarded in increasing weight and again the connected components are merged until a single component is left.

4.2.3 General algorithmic setup

We ran every algorithm 50 times on each problem and in any single run we chose to allow a maximum of $20 \cdot 10^3$ evaluations for the real–valued problems of dimensionality $l = 10$ and the binary problems of dimensionality $l = 100$ and a maximum of $100 \cdot 10^3$ evaluations for the real–valued problems of dimensionality $l = 100$ and the binary problems of dimensionality $l = 1000$. As a result of imposing the restriction of a maximum of evaluations, a value for the population size n exists for each MOEA such that the MOEA will perform best. For too large population sizes, the search will move towards a random search and for too small population sizes, there is not enough information to perform adequate model selection and induction. We therefore increased the population size in steps of 25 to find the best results. To actually select the best population size, we selected the result with the lowest value for the $\boldsymbol{D}_{\boldsymbol{\mathcal{P}_F} \rightarrow \mathcal{S}}$ indicator.

4.2.4 Algorithms

We tested a few variants of three MOEAs. In the following we will describe the details that are required in addition to the details given in earlier sections for constructing the actual MOEAs that we will use for testing.

<u>*SPEA*</u>
For SPEA, we used uniform crossover and one–point crossover with a probability of 0.8. Bit flipping mutation was used in combination with either of these recombination operators with a probability of 0.01. These settings were used previously by the SPEA authors [10]. We allowed the size of the external storage in SPEA to become as large as the population size. For the real problems, we encoded every variable with 30 bits.

NSGA–II
For NSGA–II, we used the same crossover and mutation operators and the same encoding for the real variables.

$\mathbb{MIDEA}$
For $\mathbb{MIDEA}$, we used the leader clustering algorithm in the objective space such that four clusters were constructed on average. If the number of clusters becomes too large, the requirements for the population size increases in order to facilitate proper factorization selection in each cluster. We do not suggest that the number of clusters we use is optimal, but it will serve to indicate the effectiveness of parallel exploration along the Pareto front as well as diversity preservation. In each cluster, we either used the univariate factorization or we estimated a Bayesian factorization in the case of real variables. However, in the case of 100–dimensional real–valued problems, we allowed only at most a single parent for any variable. In the case of binary variables, we used the optimal dependency tree algorithm by Chow and Liu [2] to estimate a tree factorization in each cluster. To further investigate the influence of the different components in the $\mathbb{MIDEA}$ algorithm, we also performed tests in which only a single cluster is used. Furthermore, we also replaced the use of estimating probability distributions by the use of one–point crossover and uniform crossover with mutation as used in the SPEA and NSGA–II algorithms. In the case of clustering in combination with the use of crossover operators, restricted mating was employed in order to ensure clustered exploration along the front. In restricted mating crossover, an offspring is produced using two parent solutions that are picked from the same cluster. For the truncation percentile, we used the rule of thumb by Mühlenbein and Mahnig [9] and set τ to 0.3. Furthermore, we set the diversity preservation parameter to $\delta = 1.5$.

5 Results

For each of the metrics, we computed their average and standard deviation over the 50 runs to get an assessment of their performance. The averages are tabulated in Figures 3, 4, 7, 8, 11 and 12. The best results are written in boldface. For each algorithm, the type of recombination is indicated as a superscript. The $\mathbb{MIDEA}$ algorithms are indicated by a single $\mathbb{M}$ symbol. For all tested $\mathbb{MIDEA}$ algorithms, the subscript indicates whether only a single cluster was used. Without a subscript, the leader algorithm was used in the objective space. The population sizes that led to the best performance, are tabulated in Figures 15 and 16. Although the average behavior is the most interesting, the standard deviations are vital to determine whether the differences in the average behavior of the different algorithms are significant The standard deviations are tabulated in Figures 5, 6, 9, 10, 13 and 14. In addition, we have performed Aspin–Welch–Satterthwaite (AWS) statistical hypothesis T–tests at a significance level of $\alpha = 0.05$. The AWS T–test is a statistical hypothesis test for the equality of means in which the equality of variances is not assumed. For each problem, we statistically verified for each pair of algorithms whether the average obtained metric values differ significantly. We assigned a value of 1 if an algorithm scored significantly better and a value of -1 if an algorithm scored significantly worse. We summed the so obtained matrices over all problems to get the statistically significant improvement matrices that are shown in figures 17 through 19. We also computed the sum for each algorithm of its significant improvement values over all other algorithms to indicate the summed relative statistically significant performance of the algorithms.

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Average Front Distance **AFD**								
EA	BT_1^{10}	ZDT_4^{10}	ZDT_6^{10}	CTP_7^{10}	BT_1^{100}	ZDT_4^{100}	ZDT_6^{100}	CTP_7^{100}
SPEA^{UX}	$100{\cdot}10^5$	4.62	0.193	7.97	$100{\cdot}10^5$	470	7.64	499
SPEA^{1X}	$100{\cdot}10^5$	3.90	0.172	7.31	$100{\cdot}10^5$	447	7.06	476
$\text{NSGA-II}^{\text{UX}}$	$100{\cdot}10^5$	4.39	0.303	7.25	$100{\cdot}10^5$	360	5.99	348
$\text{NSGA-II}^{\text{1X}}$	$100{\cdot}10^5$	**1.40**	0.328	**3.32**	$100{\cdot}10^5$	297	6.59	303
$\mathbb{M}_{\text{1 Cluster}}^{\text{UX}}$	$100{\cdot}10^5$	4.43	0.358	6.63	$100{\cdot}10^5$	374	6.72	378
$\mathbb{M}_{\text{1 Cluster}}^{\text{1X}}$	$100{\cdot}10^5$	1.89	0.291	4.13	$100{\cdot}10^5$	336	6.81	345
$\mathbb{M}^{\text{UX}}$	$100{\cdot}10^5$	3.98	0.354	7.27	$100{\cdot}10^5$	311	5.96	326
$\mathbb{M}^{\text{1X}}$	$100{\cdot}10^5$	2.03	0.311	3.95	$100{\cdot}10^5$	328	6.74	335
$\mathbb{M}_{\text{1 Cluster}}^{\text{Univariate}}$	$100{\cdot}10^5$	14.0	1.08	16.5	$100{\cdot}10^5$	774	3.06	875
$\mathbb{M}_{\text{1 Cluster}}^{\text{Learning}}$	$100{\cdot}10^5$	11.2	**0.00239**	15.3	$100{\cdot}10^5$	597	**0.434**	600
$\mathbb{M}^{\text{Univariate}}$	$100{\cdot}10^5$	5.00	0.306	8.64	$100{\cdot}10^5$	157	4.60	**161**
$\mathbb{M}^{\text{Learning}}$	$\mathbf{998{\cdot}10^4}$	11.5	0.287	12.6	$\mathbf{100{\cdot}10^5}$	**144**	1.30	165

Figure 3: Average of the **AFD** metric on all real–valued problems.

Average Front Distance **AFD**								
EA	MS^{100}	KN^{100}	SC^{100}	MST^{105}	MS^{1000}	KN^{1000}	SC^{1000}	MST^{1035}
SPEA^{UX}	6.10	5.14	2.93	1.31	147	67.3	442	3.39
SPEA^{1X}	7.15	5.14	2.99	1.39	237	83.1	376	3.09
$\text{NSGA-II}^{\text{UX}}$	**4.52**	**4.22**	**1.79**	1.09	147	56.1	185	3.55
$\text{NSGA-II}^{\text{1X}}$	8.03	5.40	2.64	1.36	250	90.5	254	3.18
$\mathbb{M}_{\text{1 Cluster}}^{\text{UX}}$	8.18	5.62	2.13	1.54	194	76.5	241	4.37
$\mathbb{M}_{\text{1 Cluster}}^{\text{1X}}$	10.5	6.65	2.49	1.52	326	136	364	3.96
$\mathbb{M}^{\text{UX}}$	9.01	5.01	2.12	**0.906**	213	59.3	254	4.63
$\mathbb{M}^{\text{1X}}$	12.7	6.17	2.69	1.07	359	146	410	4.32
$\mathbb{M}_{\text{1 Cluster}}^{\text{Univariate}}$	8.35	9.95	1.92	1.95	124	105	**85.9**	5.58
$\mathbb{M}_{\text{1 Cluster}}^{\text{Learning}}$	16.5	6.52	2.39	1.82	147	73.1	136	2.90
$\mathbb{M}^{\text{Univariate}}$	9.07	5.34	2.01	1.15	**26.5**	**22.3**	129	2.41
$\mathbb{M}^{\text{Learning}}$	16.5	7.23	5.38	1.39	113	64.0	472	**2.13**

Figure 4: Average of the **AFD** metric on all combinatorial problems.

Average Front Distance **AFD**								
EA	BT_1^{10}	ZDT_4^{10}	ZDT_6^{10}	CTP_7^{10}	BT_1^{100}	ZDT_4^{100}	ZDT_6^{100}	CTP_7^{100}
SPEA^{UX}	161	2.15	0.0208	2.64	0.0510	36.4	1.08	45.7
SPEA^{1X}	458	1.64	0.0226	3.12	0.0790	30.0	0.101	42.4
$\text{NSGA-II}^{\text{UX}}$	831	1.99	0.0383	1.59	0.484	32.8	0.0791	10.8
$\text{NSGA-II}^{\text{1X}}$	$138{\cdot}10^1$	0.789	0.0411	1.43	0.484	21.3	0.0788	11.0
$\mathbb{M}_{\text{1 Cluster}}^{\text{UX}}$	$455{\cdot}10^1$	1.54	0.0641	2.05	0.125	32.3	0.149	25.3
$\mathbb{M}_{\text{1 Cluster}}^{\text{1X}}$	$229{\cdot}10^1$	0.876	0.0414	1.97	0.125	18.4	0.145	21.2
$\mathbb{M}^{\text{UX}}$	$187{\cdot}10^1$	1.77	0.101	3.14	0.125	23.5	0.132	29.9
$\mathbb{M}^{\text{1X}}$	$167{\cdot}10^1$	1.00	0.0768	1.32	0.125	21.5	0.134	25.7
$\mathbb{M}_{\text{1 Cluster}}^{\text{Univariate}}$	324	7.76	0.379	7.78	0.217	139	1.40	134
$\mathbb{M}_{\text{1 Cluster}}^{\text{Learning}}$	$119{\cdot}10^1$	5.71	0.00114	6.47	0.280	93.8	0.184	62.1
$\mathbb{M}^{\text{Univariate}}$	$478{\cdot}10^1$	4.16	0.212	6.62	0.177	74.6	0.272	45.0
$\mathbb{M}^{\text{Learning}}$	$320{\cdot}10^2$	6.29	0.218	5.69	0.250	23.2	1.68	43.3

Figure 5: Standard deviations of the **AFD** metric on all real–valued problems.

Average Front Distance **AFD**								
EA	MS^{100}	KN^{100}	SC^{100}	MST^{105}	MS^{1000}	KN^{1000}	SC^{1000}	MST^{1035}
SPEA^{UX}	3.61	1.24	1.30	0.360	13.4	6.77	46.8	0.435
SPEA^{1X}	3.14	0.714	1.00	0.294	20.9	5.95	27.7	0.511
$\text{NSGA-II}^{\text{UX}}$	2.22	0.801	0.384	0.276	12.3	6.70	16.4	0.443
$\text{NSGA-II}^{\text{1X}}$	2.79	0.605	0.392	0.280	15.5	3.91	22.7	0.380
$\mathbb{M}_{\text{1 Cluster}}^{\text{UX}}$	3.73	0.798	0.496	0.233	7.20	6.19	35.4	0.382
$\mathbb{M}_{\text{1 Cluster}}^{\text{1X}}$	3.11	1.17	0.459	0.228	17.9	9.77	21.7	0.522
$\mathbb{M}^{\text{UX}}$	3.82	0.863	0.481	0.133	12.8	5.62	28.9	0.454
$\mathbb{M}^{\text{1X}}$	3.97	1.20	0.571	0.209	16.4	7.19	30.7	0.293
$\mathbb{M}_{\text{1 Cluster}}^{\text{Univariate}}$	1.89	0.936	0.373	0.190	6.09	5.28	6.58	0.454
$\mathbb{M}_{\text{1 Cluster}}^{\text{Learning}}$	2.33	1.12	0.547	0.190	5.94	7.36	15.8	0.486
$\mathbb{M}^{\text{Univariate}}$	2.94	0.962	0.702	0.215	6.98	3.52	57.5	0.299
$\mathbb{M}^{\text{Learning}}$	4.66	1.30	2.26	0.286	37.7	21.6	322	0.134

Figure 6: Standard deviations of the **AFD** metric on all combinatorial problems.

Front Spread **FS**								
EA	BT_1^{10}	ZDT_4^{10}	ZDT_6^{10}	CTP_7^{10}	BT_1^{100}	ZDT_4^{100}	ZDT_6^{100}	CTP_7^{100}
SPEA^{UX}	225	51.4	5.22	44.9	2.06	692	1.85	733
SPEA^{1X}	369	55.8	5.26	46.3	2.31	736	3.02	773
$\text{NSGA-II}^{\text{UX}}$	179	3.60	1.09	1.76	0.413	35.2	0.756	29.3
$\text{NSGA-II}^{\text{1X}}$	23.4	8.93	1.03	1.31	1.02	33.4	0.665	13.9
$\mathbb{M}^{\text{UX}}_{\text{1 Cluster}}$	655	8.55	2.90	39.1	2.18	395	3.43	365
$\mathbb{M}^{\text{1X}}_{\text{1 Cluster}}$	78.6	2.46	1.92	1.41	2.27	94.0	1.40	88.6
$\mathbb{M}^{\text{UX}}$	685	40.8	4.11	41.8	2.15	740	4.75	737
$\mathbb{M}^{\text{1X}}$	262	3.38	3.94	58.9	2.29	359	2.30	371
$\mathbb{M}^{\text{Univariate}}_{\text{1 Cluster}}$	293	70.8	1.15	84.7	1.82	393	0.180	347
$\mathbb{M}^{\text{Learning}}_{\text{1 Cluster}}$	$129 \cdot 10^1$	84.9	3.00	87.4	2.12	635	2.20	342
$\mathbb{M}^{\text{Univariate}}$	$209 \cdot 10^1$	90.4	**5.29**	114	2.45	636	**8.10**	619
$\mathbb{M}^{\text{Learning}}$	$\mathbf{164 \cdot 10^2}$	**197**	3.68	**188**	**3.28**	$\mathbf{175 \cdot 10^1}$	3.97	$\mathbf{183 \cdot 10^1}$

Figure 7: Average of the **FS** metric on all real–valued problems.

Front Spread **FS**								
EA	MS^{100}	KN^{100}	SC^{100}	MST^{105}	MS^{1000}	KN^{1000}	SC^{1000}	MST^{1035}
SPEA^{UX}	109	69.5	**64.6**	30.6	288	254	631	52.1
SPEA^{1X}	123	82.6	51.0	32.5	399	308	**636**	50.8
$\text{NSGA-II}^{\text{UX}}$	110	71.8	16.1	26.3	370	288	144	33.7
$\text{NSGA-II}^{\text{1X}}$	129	76.6	12.8	23.9	364	291	107	36.6
$\mathbb{M}^{\text{UX}}_{\text{1 Cluster}}$	114	82.0	19.6	19.8	389	347	164	37.1
$\mathbb{M}^{\text{1X}}_{\text{1 Cluster}}$	122	81.4	17.3	21.5	421	301	154	44.3
$\mathbb{M}^{\text{UX}}$	173	114	21.3	**37.3**	**706**	**585**	319	**77.3**
$\mathbb{M}^{\text{1X}}$	**169**	98.9	20.5	32.2	681	521	249	76.4
$\mathbb{M}^{\text{Univariate}}_{\text{1 Cluster}}$	68.3	39.4	16.2	17.4	111	106	11.9	22.7
$\mathbb{M}^{\text{Learning}}_{\text{1 Cluster}}$	123	83.8	18.4	17.8	153	199	142	36.9
$\mathbb{M}^{\text{Univariate}}$	167	**108**	23.8	26.3	559	485	168	55.1
$\mathbb{M}^{\text{Learning}}$	162	104	20.6	24.1	579	544	143	46.2

Figure 8: Average of the **FS** metric on all combinatorial problems.

Front Spread **FS**								
EA	BT_1^{10}	ZDT_4^{10}	ZDT_6^{10}	CTP_7^{10}	BT_1^{100}	ZDT_4^{100}	ZDT_6^{100}	CTP_7^{100}
SPEA^{UX}	162	15.5	0.343	11.0	0.107	168	1.62	155
SPEA^{1X}	459	12.0	0.471	16.4	0.128	179	0.703	191
$\text{NSGA-II}^{\text{UX}}$	291	3.60	0.0567	0.839	0.631	13.9	0.0446	17.4
$\text{NSGA-II}^{\text{1X}}$	97.7	30.4	0.0865	0.793	1.43	10.0	0.147	3.82
$\mathbb{M}^{\text{UX}}_{\text{1 Cluster}}$	$183{\cdot}10^1$	9.07	1.28	24.3	0.108	110	0.698	99.2
$\mathbb{M}^{\text{1X}}_{\text{1 Cluster}}$	337	0.74	1.11	0.457	0.274	45.1	0.577	94.4
$\mathbb{M}^{\text{UX}}$	$179{\cdot}10^1$	24.6	1.14	23.4	0.154	138	0.576	105
$\mathbb{M}^{\text{1X}}$	991	1.39	1.27	54.7	0.251	105	0.596	47.4
$\mathbb{M}^{\text{Univariate}}_{\text{1 Cluster}}$	281	49.5	1.24	49.6	0.381	259	0.307	242
$\mathbb{M}^{\text{Learning}}_{\text{1 Cluster}}$	$121{\cdot}10^1$	54.5	1.93	49.4	0.603	374	2.50	359
$\mathbb{M}^{\text{Univariate}}$	$443{\cdot}10^1$	57.3	1.42	43.9	0.375	205	0.782	261
$\mathbb{M}^{\text{Learning}}$	$321{\cdot}10^2$	42.2	1.93	43.8	0.706	233	3.57	224

Figure 9: Standard deviations of the **FS** metric on all real–valued problems.

Front Spread **FS**								
EA	MS^{100}	KN^{100}	SC^{100}	MST^{105}	MS^{1000}	KN^{1000}	SC^{1000}	MST^{1035}
SPEA^{UX}	16.6	8.88	33.2	4.80	38.4	28.6	255	6.12
SPEA^{1X}	17.8	8.77	16.3	4.19	50.2	37.7	294	4.81
$\text{NSGA-II}^{\text{UX}}$	14.3	10.1	4.73	4.05	34.3	29.1	34.0	5.39
$\text{NSGA-II}^{\text{1X}}$	9.86	14.5	1.57	3.64	50.6	24.5	31.3	6.84
$\mathbb{M}^{\text{UX}}_{\text{1 Cluster}}$	15.7	8.98	5.07	2.34	52.3	22.8	39.5	6.85
$\mathbb{M}^{\text{1X}}_{\text{1 Cluster}}$	13.8	9.46	5.19	3.44	86.6	53.2	30.3	4.84
$\mathbb{M}^{\text{UX}}$	16.0	7.61	6.05	7.05	46.8	26.7	73.0	11.3
$\mathbb{M}^{\text{1X}}$	13.6	10.3	5.69	6.13	28.1	45.2	57.2	8.32
$\mathbb{M}^{\text{Univariate}}_{\text{1 Cluster}}$	11.9	8.20	4.00	1.54	16.1	16.9	7.25	4.09
$\mathbb{M}^{\text{Learning}}_{\text{1 Cluster}}$	14.0	9.31	4.70	1.96	11.6	29.8	45.8	3.21
$\mathbb{M}^{\text{Univariate}}$	13.7	8.59	5.65	4.16	37.1	34.7	108	3.61
$\mathbb{M}^{\text{Learning}}$	19.2	11.7	7.88	3.69	76.4	79.1	95.6	6.87

Figure 10: Standard deviations of the **FS** metric on all combinatorial problems.

Front Occupation **FO**								
EA	BT_1^{10}	ZDT_4^{10}	ZDT_6^{10}	CTP_7^{10}	BT_1^{100}	ZDT_4^{100}	ZDT_6^{100}	CTP_7^{100}
SPEA^{UX}	**60.9**	99.0	50.0	43.5	49.8	27.6	18.7	26.7
SPEA^{1X}	38.7	**187**	49.6	43.2	48.8	27.4	29.3	26.8
$\text{NSGA-II}^{\text{UX}}$	5.42	59.7	47.5	**59.3**	100	5.80	6.00	4.00
$\text{NSGA-II}^{\text{1X}}$	29.5	32.7	31.2	9.98	75.0	5.00	6.60	3.00
$\mathbb{M}_{\text{1 Cluster}}^{\text{UX}}$	9.92	41.7	8.06	9.00	14.4	12.8	14.4	12.6
$\mathbb{M}_{\text{1 Cluster}}^{\text{1X}}$	13.4	30.3	6.52	11.9	16.5	7.10	6.64	5.94
$\mathbb{M}^{\text{UX}}$	13.9	10.0	8.48	8.62	19.1	20.0	19.6	21.7
$\mathbb{M}^{\text{1X}}$	9.94	31.4	7.32	15.6	17.4	12.2	9.76	12.2
$\mathbb{M}_{\text{1 Cluster}}^{\text{Univariate}}$	5.74	6.88	4.90	4.14	36.7	6.9	2.55	3.20
$\mathbb{M}_{\text{1 Cluster}}^{\text{Learning}}$	6.06	8.36	**258**	4.96	13.1	5.25	**369**	3.75
$\mathbb{M}^{\text{Univariate}}$	12.5	68.7	56.3	34.0	64.5	**106**	27.7	**78.9**
$\mathbb{M}^{\text{Learning}}$	30.1	26.4	197	32.1	**111**	50.8	163	43.0

Figure 11: Average of the **FO** metric on all real–valued problems.

Front Occupation **FO**								
EA	MS^{100}	KN^{100}	SC^{100}	MST^{105}	MS^{1000}	KN^{1000}	SC^{1000}	MST^{1035}
SPEA^{UX}	96.4	49.8	27.2	49.6	49.4	49.5	26.2	50.0
SPEA^{1X}	96.4	87.6	27.7	124	49.9	49.7	26.5	98.6
$\text{NSGA-II}^{\text{UX}}$	200	**175**	150	200	35.9	33.1	7.50	250
$\text{NSGA-II}^{\text{1X}}$	99.9	**175**	**212**	200	42.9	37.0	7.20	249
$\mathbb{M}_{\text{1 Cluster}}^{\text{UX}}$	32.2	18.4	7.80	63.8	17.6	16.6	4.40	14.4
$\mathbb{M}_{\text{1 Cluster}}^{\text{1X}}$	41.9	28.7	10.3	62.2	18.9	15.3	5.70	21.8
$\mathbb{M}^{\text{UX}}$	73.3	30.2	11.8	61.1	28.5	27.6	6.7	24.8
$\mathbb{M}^{\text{1X}}$	71.3	62.9	17.2	352	34.4	18.5	5.90	27.2
$\mathbb{M}_{\text{1 Cluster}}^{\text{Univariate}}$	111	14.3	10.2	105	39.3	16.4	28.4	15.7
$\mathbb{M}_{\text{1 Cluster}}^{\text{Learning}}$	227	34.7	13.5	42.8	19.9	40.7	6.43	286
$\mathbb{M}^{\text{Univariate}}$	132	38.1	19.3	**299**	108	57.2	**255**	41.6
$\mathbb{M}^{\text{Learning}}$	**390**	164	34.9	93.5	**548**	**67.0**	30.3	$\mathbf{105 \cdot 10^1}$

Figure 12: Average of the **FO** metric on all combinatorial problems.

Front Occupation **FO**								
EA	BT_1^{10}	ZDT_4^{10}	ZDT_6^{10}	CTP_7^{10}	BT_1^{100}	ZDT_4^{100}	ZDT_6^{100}	CTP_7^{100}
SPEA^{UX}	60.9	4.72	0.140	4.19	0.433	1.46	15.3	0.943
SPEA^{1X}	38.7	24.3	0.592	5.63	1.85	1.26	1.49	0.789
$\text{NSGA-II}^{\text{UX}}$	5.42	43.9	8.15	31.7	0.00	2.71	1.41	1.79
$\text{NSGA-II}^{\text{1X}}$	29.5	14.8	5.59	4.73	0.00	1.48	1.36	1.26
$\mathbb{M}_{\text{1 Cluster}}^{\text{UX}}$	9.92	10.3	1.69	2.26	2.59	2.87	3.44	3.06
$\mathbb{M}_{\text{1 Cluster}}^{\text{1X}}$	13.3	10.6	1.40	5.02	2.62	2.30	1.69	2.73
$\mathbb{M}^{\text{UX}}$	13.9	2.00	1.59	1.85	2.91	3.77	3.77	3.52
$\mathbb{M}^{\text{1X}}$	9.94	14.3	1.42	5.11	2.55	3.21	2.46	2.25
$\mathbb{M}_{\text{1 Cluster}}^{\text{Univariate}}$	5.74	8.99	2.42	1.36	23.9	1.48	2.65	1.21
$\mathbb{M}_{\text{1 Cluster}}^{\text{Learning}}$	6.06	7.20	120	1.45	2.45	2.39	256	1.51
$\mathbb{M}^{\text{Univariate}}$	12.5	36.2	32.7	26.4	23.8	34.3	4.57	53.3
$\mathbb{M}^{\text{Learning}}$	30.1	23.0	55.3	17.4	7.85	7.41	141	5.87

Figure 13: Standard deviations of the **FO** metric on all real–valued problems.

Front Occupation **FO**								
EA	MS^{100}	KN^{100}	SC^{100}	MST^{105}	MS^{1000}	KN^{1000}	SC^{1000}	MST^{1035}
SPEA^{UX}	3.73	0.663	2.16	1.30	1.43	0.780	1.30	0.196
SPEA^{1X}	4.62	6.13	2.57	3.21	0.310	0.693	1.99	2.69
$\text{NSGA-II}^{\text{UX}}$	0.00	0.00	0.00	0.00	6.20	5.05	2.38	1.96
$\text{NSGA-II}^{\text{1X}}$	0.300	0.00	76.4	0.00	8.26	6.90	2.71	3.48
$\mathbb{M}_{\text{1 Cluster}}^{\text{UX}}$	4.32	2.42	1.84	9.05	1.93	1.43	0.800	2.42
$\mathbb{M}_{\text{1 Cluster}}^{\text{1X}}$	4.67	4.31	2.85	13.3	3.26	1.90	1.61	3.28
$\mathbb{M}^{\text{UX}}$	9.42	5.00	8.36	22.7	3.08	2.29	1.68	3.76
$\mathbb{M}^{\text{1X}}$	5.07	18.1	15.4	124	3.16	0.806	1.58	3.34
$\mathbb{M}_{\text{1 Cluster}}^{\text{Univariate}}$	56.4	2.24	2.95	13.7	8.14	1.92	46.8	2.19
$\mathbb{M}_{\text{1 Cluster}}^{\text{Learning}}$	73.6	5.45	4.29	14.4	3.87	1.48	1.68	364
$\mathbb{M}^{\text{Univariate}}$	17.5	12.7	36.0	212	13.6	3.94	171	12.2
$\mathbb{M}^{\text{Learning}}$	38.2	122	118	27.1	39.0	8.22	59.4	49.8

Figure 14: Standard deviations of the **FO** metric on all combinatorial problems.

Population Size n								
EA	BT_1^{10}	ZDT_4^{10}	ZDT_6^{10}	CTP_7^{10}	BT_1^{100}	ZDT_4^{100}	ZDT_6^{100}	CTP_7^{100}
SPEA$^{\text{UX}}$	50	50	25	25	25	25	25	25
SPEA$^{\text{1X}}$	25	100	25	25	25	25	25	25
NSGA-II$^{\text{UX}}$	200	200	100	100	100	200	200	150
NSGA-II$^{\text{1X}}$	200	375	75	300	75	200	150	300
$\mathbb{M}^{\text{UX}}_{\text{1 Cluster}}$	75	100	25	25	100	125	200	125
$\mathbb{M}^{\text{1X}}_{\text{1 Cluster}}$	100	450	25	300	125	325	100	175
$\mathbb{M}^{\text{UX}}$	225	25	25	25	125	200	200	300
$\mathbb{M}^{\text{1X}}$	150	475	25	725	125	200	100	150
$\mathbb{M}^{\text{Univariate}}_{\text{1 Cluster}}$	150	50	75	50	100	75	375	50
$\mathbb{M}^{\text{Learning}}_{\text{1 Cluster}}$	150	75	425	75	175	100	700	100
$\mathbb{M}^{\text{Univariate}}$	275	125	200	125	250	200	800	200
$\mathbb{M}^{\text{Learning}}$	450	200	250	150	225	300	400	250

Figure 15: Population sizes used for the real–valued problems.

Population Size n								
EA	MS^{100}	KN^{100}	SC^{100}	MST^{105}	MS^{1000}	KN^{1000}	SC^{1000}	MST^{1035}
SPEA$^{\text{UX}}$	50	25	25	25	25	25	25	25
SPEA$^{\text{1X}}$	50	50	25	100	25	25	25	50
NSGA-II$^{\text{UX}}$	200	175	150	200	200	250	200	250
NSGA-II$^{\text{1X}}$	100	175	250	200	150	200	150	250
$\mathbb{M}^{\text{UX}}_{\text{1 Cluster}}$	100	150	225	1250	175	200	100	350
$\mathbb{M}^{\text{1X}}_{\text{1 Cluster}}$	100	175	125	1200	150	150	150	375
$\mathbb{M}^{\text{UX}}$	175	175	100	950	175	250	125	450
$\mathbb{M}^{\text{1X}}$	150	175	150	675	200	125	100	425
$\mathbb{M}^{\text{Univariate}}_{\text{1 Cluster}}$	325	250	350	2700	650	500	150	475
$\mathbb{M}^{\text{Learning}}_{\text{1 Cluster}}$	425	400	500	1800	1000	700	500	1500
$\mathbb{M}^{\text{Univariate}}$	250	200	400	900	475	600	475	775
$\mathbb{M}^{\text{Learning}}$	500	450	850	1950	750	1250	1900	1250

Figure 16: Population sizes used for the combinatorial problems.

Average Front Distance **AFD**													
Statistically Significant Improvement Matrix	SPEA^{UX}	SPEA^{1X}	NSGA-II^{UX}	NSGA-II^{1X}	$\mathbb{M}^{UX}_{1\ Cluster}$	$\mathbb{M}^{1X}_{1\ Cluster}$	$\mathbb{M}^{UX}$	$\mathbb{M}^{1X}$	$\mathbb{M}^{Univariate}_{1\ Cluster}$	$\mathbb{M}^{Learning}_{1\ Cluster}$	$\mathbb{M}^{Univariate}$	$\mathbb{M}^{Learning}$	**Sum**
SPEA^{UX}	0	-6	-8	-2	1	-1	-4	-2	4	1	-9	-1	-27
SPEA^{1X}	6	0	-8	-3	-3	1	-4	2	6	-1	-9	-1	-14
NSGA-II^{UX}	8	8	0	3	11	5	4	3	7	3	-7	1	46
NSGA-II^{1X}	2	3	-3	0	3	10	-2	9	4	-2	-9	-2	13
$\mathbb{M}^{UX}_{1\ Cluster}$	-1	3	-11	-3	0	0	-5	-1	5	1	-8	-4	-24
$\mathbb{M}^{1X}_{1\ Cluster}$	1	-1	-5	-10	0	0	-5	0	2	-2	-11	-3	-34
$\mathbb{M}^{UX}$	4	4	-4	2	5	5	0	4	4	3	-8	0	19
$\mathbb{M}^{1X}$	2	-2	-3	-9	1	0	-4	0	2	-5	-10	-2	-30
$\mathbb{M}^{Univariate}_{1\ Cluster}$	-4	-6	-7	-4	-5	-2	-4	-2	0	-7	-10	-9	-60
$\mathbb{M}^{Learning}_{1\ Cluster}$	-1	1	-3	2	-1	2	-3	5	7	0	-9	-3	-3
$\mathbb{M}^{Univariate}$	9	9	7	9	8	11	8	10	10	9	0	5	95
$\mathbb{M}^{Learning}$	1	1	-1	2	4	3	0	2	9	3	-5	0	19

Figure 17: Number of times an improvement was found to be statistically significant in the AFD metric, summed over all tested problems. The numbers in a single row indicate the summed number of significantly better or worse results compared to the algorithms in the different columns.

Front Spread **FS**													
Statistically Significant Improvement Matrix	SPEA^{UX}	SPEA^{1X}	NSGA-II^{UX}	NSGA-II^{1X}	$\mathbb{M}^{UX}_{1\ Cluster}$	$\mathbb{M}^{1X}_{1\ Cluster}$	$\mathbb{M}^{UX}$	$\mathbb{M}^{1X}$	$\mathbb{M}^{Univariate}_{1\ Cluster}$	$\mathbb{M}^{Learning}_{1\ Cluster}$	$\mathbb{M}^{Univariate}$	$\mathbb{M}^{Learning}$	**Sum**
SPEA^{UX}	0	-7	9	8	3	5	-4	0	11	3	-6	-6	16
SPEA^{1X}	7	0	16	14	8	11	-2	2	11	7	-5	-5	64
NSGA-II^{UX}	-9	-16	0	5	-12	-6	-15	-14	3	-9	-14	-13	-100
NSGA-II^{1X}	-8	-14	-5	0	-10	-7	-16	-14	1	-7	-16	-15	-111
$\mathbb{M}^{UX}_{1\ Cluster}$	-3	-8	12	10	0	4	-12	-8	9	0	-15	-13	-24
$\mathbb{M}^{1X}_{1\ Cluster}$	-5	-11	6	7	-4	0	-14	-14	5	-3	-15	-14	-62
$\mathbb{M}^{UX}$	4	2	15	16	12	14	0	7	11	9	1	1	92
$\mathbb{M}^{1X}$	0	-2	14	14	8	14	-7	0	9	5	-5	-4	46
$\mathbb{M}^{Univariate}_{1\ Cluster}$	-11	-11	-3	-1	-9	-5	-11	-9	0	-12	-15	-16	-103
$\mathbb{M}^{Learning}_{1\ Cluster}$	-3	-7	9	7	0	3	-9	-5	12	0	-12	-13	-18
$\mathbb{M}^{Univariate}$	6	5	14	16	15	15	-1	5	15	12	0	-2	100
$\mathbb{M}^{Learning}$	6	5	13	15	13	14	-1	4	16	13	2	0	100

Figure 18: Number of times an improvement was found to be statistically significant in the FS metric, summed over all tested problems. The numbers in a single row indicate the summed number of significantly better or worse results compared to the algorithms in the different columns.

Front Occupation **FO**													
Statistically Significant Improvement Matrix	SPEA$^{\text{UX}}$	SPEA$^{\text{1X}}$	NSGA-II$^{\text{UX}}$	NSGA-II$^{\text{1X}}$	$\mathbb{M}^{\text{UX}}_{\text{1 Cluster}}$	$\mathbb{M}^{\text{1X}}_{\text{1 Cluster}}$	$\mathbb{M}^{\text{UX}}$	$\mathbb{M}^{\text{1X}}$	$\mathbb{M}^{\text{Univariate}}_{\text{1 Cluster}}$	$\mathbb{M}^{\text{Learning}}_{\text{1 Cluster}}$	$\mathbb{M}^{\text{Univariate}}$	$\mathbb{M}^{\text{Learning}}$	**Sum**
SPEA$^{\text{UX}}$	0	-3	2	4	13	14	13	12	12	8	-4	-8	63
SPEA$^{\text{1X}}$	3	0	1	3	16	16	16	14	14	8	-2	-6	83
NSGA-II$^{\text{UX}}$	-2	-1	0	1	8	8	7	5	10	3	-2	-7	30
NSGA-II$^{\text{1X}}$	-4	-3	-1	0	7	9	7	5	11	2	-6	-9	18
$\mathbb{M}^{\text{UX}}_{\text{1 Cluster}}$	-13	-16	-8	-7	0	-3	-11	-7	1	-2	-16	-13	-95
$\mathbb{M}^{\text{1X}}_{\text{1 Cluster}}$	-14	-16	-8	-9	3	0	-8	-11	3	-2	-14	-14	-90
$\mathbb{M}^{\text{UX}}$	-13	-16	-7	-7	11	8	0	1	5	1	-14	-15	-46
$\mathbb{M}^{\text{1X}}$	-12	-14	-5	-5	7	11	-1	0	8	4	-12	-12	-31
$\mathbb{M}^{\text{Univariate}}_{\text{1 Cluster}}$	-12	-14	-10	-11	-1	-3	-5	-8	0	-6	-15	-12	-97
$\mathbb{M}^{\text{Learning}}_{\text{1 Cluster}}$	-8	-8	-3	-2	2	2	-1	-4	6	0	-6	-11	-33
$\mathbb{M}^{\text{Univariate}}$	4	2	2	6	16	14	14	12	15	6	0	-4	87
$\mathbb{M}^{\text{Learning}}$	8	6	7	9	13	14	15	12	12	11	4	0	111

Figure 19: Number of times an improvement was found to be statistically significant in the FO metric, summed over all tested problems. The numbers in a single row indicate the summed number of significantly better or worse results compared to the algorithms in the different columns.