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Abstract

The investigation of metric structures in both music theory and the study of music perception requires the description of metric hierarchies generated by the notes of a piece. This paper applies a mathematical model that is based on pulses acting on the notes to infer the structural description of the metric organization of musical pieces. Furthermore, we use the model of *Inner Metric Analysis* to explain perceptual phenomena involved in the listening process. Inner Metric Analysis assigns metric weights to all note events. We show that these weights often reflect a metric hierarchy that is implied by the bar lines; hence the notes generate a hierarchy that corresponds to that of the abstract grid of the bar lines. We define such a correspondence as *metric coherence*. In addition, the model provides a precise means to describe metric conflicts intentionally introduced by the composer; these conflicts prevent metric coherence.

We present results that show how our model characterizes different forms of interaction between different lines within the texture of a composition. Moreover, in our analysis we distinguish between different levels of metric organization based on local and global information. The application of these analytic perspectives to ragtimes provides differentiated explanations for the results of listening experiments. Hence, this article demonstrates how the structural description of the metric organization of musical pieces helps to answer questions arising from cognitive perspectives and thus links both music theory and perceptual studies.

1 Introduction

This paper applies the mathematical model of *Inner Metric Analysis* to the structural description of the metric organization of musical pieces and to perceptual phenomena involved in listening processes. We address different levels of the metric organization with respect to local and global information and the interaction of different lines within the texture of a composition. The application of these analytic strategies to ragtimes provides detailed explanations for the findings obtained in an empirical experiment described by Snyder & Krumhansl (2001).

The following section gives a short introduction into the specific metric problem addressed by Inner Metric Analysis. Sections 1.2 to 1.5 provide an overview of the model's relation to music theory, its new contribution to the field of computational models and cognition, the type of meter information considered by the model, the methodological background and the evaluation method for the model. In section 2 we describe the components of the model of Inner Metric Analysis. We introduce two different types of weights, namely the metric and spectral weights. Sections 3 and 4 apply these weights to a structural description of meter in music. The discussion includes analyses of musical examples stemming from different genres illustrating a concept of *Metric Coherence* in section 3. The comparison of the metric and spectral weights in section 4 conveys insights into the interaction of local and global aspects of the metric organization of musical pieces. The application of the model to the genre of ragtimes in section 5 provides in-depth explanations concerning the results of an experiment described by Snyder & Krumhansl (2001) addressing a tapping task for listeners. Section 8 provides a glossary listing key terms of the model and mathematical symbols. The appendix lists the analyses of all ragtime pieces used in the experiment by Snyder & Krumhansl (2001).

1.1 The metric problem addressed by Inner Metric Analysis

The complexity of the metric structure of music is a challenging research topic in both music theory and investigations concerning music perception and cognition. While music theorists analyze musical scores attempting to determine the characteristic features that are responsible for creating a metric hierarchy, empirical studies focus on the metric structure experienced while listening to the music. Though the perception of metric phenomena is obviously linked to the structures addressed by music theorists, the methods and results of these two research directions differ to a great extent. The in-depth structural analyses in music theory often lack a connection to perceptual questions. Empirical studies, on the other hand, often do not reach the complexity of metric phenomena investigated by theorists. This article describes a mathematical model for metric analysis that helps to link the strengths of music theoretic approaches to those of the experimental investigations concerning metric structures for the benefit of research in both directions. We demonstrate the model's potential to explain complex metric situations on a structural level and its contribution to the understanding of perceptual tasks investigated in listening experiments.

In this paper we show that a computational model for abstracting metric hierarchies can help explain the perceived metric schema in music. The ease or difficulty with which listeners can formulate this schema is reflected, for instance, in their ability to tap to the music. The main question addressed in the application of *Inner Metric Analysis* (Fleischer et al., 2000; Nestke & Noll, 2001; Fleischer, 2002; Mazzola, 2002; Fleischer, 2003; Volk, 2003) in this paper can be paraphrased as follows. What distinguishes the metric structure of, for instance, pieces by the minimalist Anton Webern and ragtimes, such as Webern's op. 27 mvmt. 2 and Scott Joplin's *Nonpareil Rag*? A simple observation is that tapping along Webern's piece is much more difficult in comparison to tapping along the ragtime. This tapping task often investigated in empirical studies points to a difference in the metric structure of these pieces which is not reflected by the metric structure implied by the bar lines. Both the Webern and Joplin pieces are notated as 2/4. This leads to the question as to how we can describe the structural difference between these pieces?

When listening to Webern's op. 27 mvmt. 2 it is difficult to perceive the metric accent schema associated with the 2/4 time signature, as discussed by Lewin (1993). Hence, the notes placed *inside* the bar lines express a different metric structure in comparison to the *outer* abstract grid of the bar lines. We call this metric structure generated by the notes inside the bars the *inner* metric structure, because it reflects the more intrinsic characteristics of the music expressed by the notes of the piece. The metric structure associated with the time signature is called the *outer* metric structure since it reflects the characteristics of a more abstract concept chosen by the composer in terms of the bar lines and the time signature.

This abstract concept can however serve as an important ingredient of the overall metric structure of a piece, such as in cases where a conflict is intentionally created between the inner and outer metric structures. Hence it is important to distinguish these two types of metric information.

The mathematical model of Inner Metric Analysis describes the inner metric structure without considering the abstract grid of the bar lines by assigning numeric metric weights to all notes, similar to the metric accents in music theory. Inner Metric Analysis generates metric hierarchies based on pulses of equally spaced notes. These metric hierarchies serve as a structural description of the piece and as an appropriate tool to address the perception of these metric hierarchies while listening to the piece.

1.2 Relations to music theoretic approaches to metric analysis

The questions addressed by Inner Metric Analysis are embedded into the contexts of studies in music theory, mathematical music theory, cognitive approaches and computational models. This section places the proposed model within the context of music theory.

Inner Metric Analysis is derived from pulse descriptions. In a similar fashion, music theoretic approaches to meter in music such as Yeston (1976), Lerdahl & Jackendoff (1983), Roeder (1994), Hasty (1997), Krebs (1999), or Cohn (2001) are based on the detection of pulses (or layers of motion). For example, Figure 1 gives a general illustration regarding the time signature 3/4 with three pulse levels, the fastest pulse consisting of eighth notes. Points at which many levels coincide are called "strong beats", while points at which few pulse levels coincide are called "weak beats". Hence, the strongest beats are the first beats of the bars (all points on the lowermost or 3/4 layer in Figure 1), followed by the second and third quarter notes of the bars (the second, third, fifth, sixth and eighth points on the middle or 1/4 layer in Figure 1). The second, fourth and sixth eighth notes are the weakest beats (every other point, starting at the second point on the uppermost or 1/8 layer in Figure 1). These alternating patterns of strong and weak beats hence create three different layers of the metric hierarchy, namely the layer of the first beats in the bars, the layer of the second and third main beats in the bars and the layer of the weaker eighth notes in between these main beats.

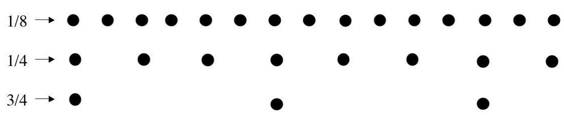

Figure 1: Metrical accent pattern of a 3/4 bar arising from the superposition of pulse layers.

Inner Metric Analysis considers different forms of superposition of pulses as will be discussed in section 2. The example of Figure 1 illustrates only one specific case of a superposition of pulses. All pulse levels have mutually dividing periods (one period is an integer multiple of another) and the same phase. From their superposition arises the typical accent pattern associated with a 3/4 time signature. Inner Metric Analysis considers all the pulses generated by the notes of the piece that can overlay with each other with very different periods and shifted at all phases. The nested pulses in Figure 1 might serve as an explanation of the metric accent pattern associated with the bar lines in any 3/4 time signature that is implied by the *abstract* grid of the bar lines. We call this type of metric schema the outer metric structure, while Inner Metric Analysis describes the inner metric structure expressed by the notes *inside* the bar lines.

Fleischer (2003) introduces the concept of metric coherence describing a correspondence between these two types of metric structures. Similar ideas underlie the concept of "metric consonance" and "metric dissonance" (Krebs, 1999). Krebs characterizes the superposition of nested pulses as in Figure 1 in a 3/4 meter as "metric consonance", because the nested pulses create a metric structure via the superposition of these pulses that reflect the typical accent hierarchy of a 3/4. Metric coherence in our sense also requires the superposition of pulses such that the typical accent hierarchy of the outer metric structure is reflected in the metric weight.

A different way of interaction of pulses taking place in a 3/4 time signature is shown in Figure 2 illustrating the situation of "metric dissonance". In the last measures of the first movement of Beethoven's *Eroica* Krebs assigns two layers of the period 3 (as multiples of quarter notes) with different phases and one layer of period 2. The example from Beethoven's *Eroica* illustrates that the actual notes may generate other pulses and hence provoke a discrepancy in comparison to the outer metric structure given by Figure 1. Moreover, this example shows that a conflict may exist between different pulses of the notes (in this case between the pulse of 2 and the pulse of 3) if their periods do not divide each other. Hence, even by ignoring the given abstract grid the metric structure of the notes reveals an appealing conflicting situation. Therefore, music theorists demand that metric accents should not be explained only in terms of abstract grids, but as arising from the concrete elements of the piece (see Hasty, 1997).

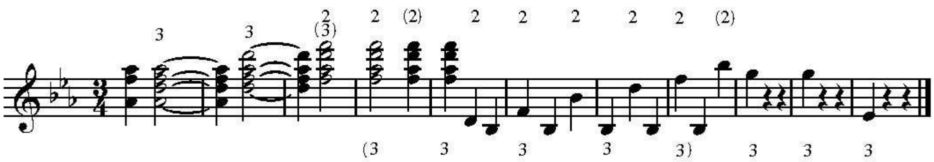

Figure 2: Three different pulses of Krebs' analysis of Beethoven's *Eroica*, 1st movement, measures 681-690.

Inner Metric Analysis realizes an objective and quantitative method of creating these metric accents by assigning to each note a metric weight based on the superposition of specific pulses evoked by the notes. The model thus realizes Krebs' idea to " ... define the meter of a work as the union of all layers of motion (i.e., series of regularly recurring pulses) active within it" (Krebs, 1999, p. 23). The computational model allows one to easily generate the metric hierarchies for entire pieces by automatically finding the pulses. By assigning metric weights to *all* notes the model is able to find metric characteristics of large sections, such as an exposition of a symphony movement, going beyond selective analyses of a small range. The examples discussed in this article demonstrate the potential of the model to objectively describe complex structural phenomena that have been addressed in music theoretic approaches. These investigations aim, for instance, at the description of the contribution of meter to the phrase structure of a piece, to the phenomenon of tension and release in music, or to the dramatic qualities of music, such as to narrate, polarize and resolve as discussed by Willner (1998), Horlacher (1992), or Cohn (1992).

1.3 Relations to the goals of computational approaches to meter perception

As a computational model Inner Metric Analysis contributes to the field of automatic extraction of meter information from music. This field is characterized by a great variety of methods, questions, goals and musical corpora involved in the evaluation of the models. In the following we briefly compare the methods and goals of some of these models in order to illustrate the new contribution of Inner Metric Analysis to this field.

The investigations in this paper using Inner Metric Analysis are concerned with the interpretation of the meter of a piece of music and to help explain perceptual phenomena, such as the ease or difficulty to tap along a piece while listening to it. Inner Metric Analysis suggests a new reference point for the evaluation of metric models that investigate these perceptual tasks. Different evaluation methods apply, for instance, to the approaches in Longuet-Higgins & Lee (1987) and Desain & Honing (1999).

Longuet-Higgins & Lee (1987) investigate the question, what criteria a listener might choose to assign a certain metric interpretation to a given sequence of notes. The investigation of this question assumes that for a given sequence of notes there exists a most plausible metric structure and that the notes themselves reveal this structure to the listener. The results of Longuet-Higgins' and Lee's paper might be evaluated by the reader who has to decide, whether a given solution indeed seems to reflect the most plausible version to him or through listening experiments.

Desain & Honing (1999) tested the further developments of Longuet-Higgins' and Lee's ideas and algorithms on a set of musical pieces (national anthems) and chose the notated bar lines as the correct answer. This approach implicates a different question: the modeling of the "right placement" of bar lines of a given piece of music. The reference point in this case is not a listener's metric interpretation of the sequence of notes, but the given bar lines of the notated score. This is counter to Krebs' recommendation that the bar lines often do not serve as the ultimate meter information about the piece.

Inner Metric Analysis demonstrates that the correct placement of bar lines and the most plausible metric interpretation while listening to the piece do not always coincide. The investigation of the mutual relation of these issues leads to intriguing questions about complex structural phenomena and their perception. Krebs (1999) describes such a complex metric situation in the beginning of Robert Schumann's *Walzer* op. 124 no. 15 (see Figure 3). Since the notated downbeats are suppressed for many measures, Krebs addresses the performance problem, that beats other than the first beats of the bars might be perceived as the actual downbeats. Hence, a perceptual model such as in Longuet-Higgins & Lee (1987) might assign different "plausible" bar lines than the notated ones (especially since the model prefers interpretations which avoid syncopation). The assumption, Schumann himself might have placed the bar lines wrongly or not in a plausible way might solve the discrepancy between the two issues. But it would ignore the competence of the composer who "worked very hard to achieve the clearest possible visual representation of the sounds that he heard in his mind" (Krebs, 1999, p. 181). The discrepancy between the actual placement of the notes and the placement of the bar lines therefore creates appealing conflicts that should be considered as an important ingredient of the metric structure.

Another example addressing the same phenomenon of questioning the absolute priority of bar lines within an experimental approach is the investigation of Drake & Palmer (1993). They address the question of how performers might emphasize metric accents while playing short musical passages. Performers were given different versions of the same note sequence with

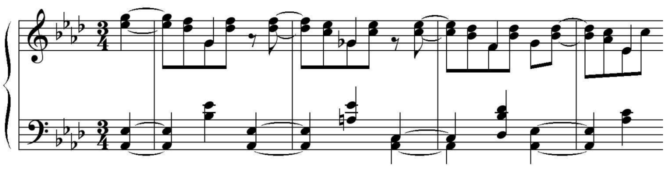

Figure 3: Robert Schumann: *Walzer*, op. 124 no. 15, mm. 1-4

different placements of bar lines, but tended to ignore these different placements while playing. They delayed in all versions the same inter-onset intervals. Hence, the actual placement of the bar lines was not the only reference point for the performers but indicates the influence of other elements of the score on the metric interpretation of the passage by the performers.[1]

Addressing these phenomena, Inner Metric Analysis offers another reference point for the evaluation of metric models that investigate perceptual tasks; not the bar lines but the inner metric structure expressed by the notes and its relation to the outer metric structure should be considered. The musical examples discussed in this article demonstrate how the investigation of this relation helps to explain complex metric situations of very different facets that point to diverse perceptual phenomena.

1.4 Types of meter information addressed by Inner Metric Analysis

Computational models of extracting meter information address different levels of complexity in the metric organization of the piece and the corresponding listening processes. Povel & Essens (1985), for example, investigated the induction of an internal clock while listening to short repetitive temporal patterns. Here not a complete metric hierarchy is addressed but the induction of a most prominent pulse or beat. Scheirer (1998) aimed at the simulation of rapid judgments listeners give about the metric structure of a piece when listening to a very short excerpt. Using signal-processing and pattern recognition tools he attempted to model the beat of a piece of music, not the complete metric hierarchy.

The metric characterization of musical pieces with Inner Metric Analysis is not directed towards the description of metric features that can be grasped very quickly by listening to extremely short examples, as in Scheirer's approach. The model addresses the richness of metric structures of very complex pieces such as symphonies or entire choral works by generating metric hierarchies. This richness can be described as a web of different interacting structure levels of rhythmic-metric characteristics similar to the different structure levels addressed in tonal studies (as in Schenkerian Theory). Different tonal levels distinguish the fixed tonal key assigned to an entire piece from local changes. These local changes can involve modulations between different key sections within the same piece and very local changes between chord progressions. All of these different levels together contribute to the complex phenomenon of *tonality* in the pitch domain.

Similarly, the description of metric structures as part of the time organization of a piece needs to address different levels. The induction of a beat as often studied in perceptual studies is only one aspect of this metric organization. The metric structure of a piece can contribute, for instance, to the phrase structure of the piece. Hence Horlacher (1992) explains

[1] This example does not imply that the bar lines never have an influence on the performance. For the investigation of the role of bar lines in piano performances see, for instance, Sloboda (1983).

the experience of the completion of a phrase in the case of Stravinksy's music with the help of metric investigations that study the nature of phrase continuity and development.

The application of Inner Metric Analysis in this article addresses the different levels of the metric structure in different ways. Two types of weights are compared, namely the metric and spectral weights, that react differently in terms of local changes or global robustness. Another method to describe the web of different structure levels is the modeling of the interaction between different voices or instrumental parts of a piece. The article therefore discusses examples where instrumental parts are extracted from the score.

Inner Metric Analysis investigates a perceptual situation that has not yet been explored extensively by cognitive studies and computational models. Describing different types of interaction between different lines of the compositional texture is linked to the following question. How does an ensemble player, knowing his own part, orientate himself within the metric structure of the entire piece to contribute best to the performance of the piece? This problem is investigated better within the tonal domain. A violinist playing single notes performs a melody. In addition these single notes can contribute to harmonic events, such as chords, that have to be coordinated with other instruments. In this case it is important for the violinist to know the interplay between his isolated part and the harmonic function of his notes in the context of other parts of the composition in order to produce the correct tone. On the contrary, for a choir singer it might not be an appropriate strategy to follow other parts while singing a tone within a cluster. This situation refers to a different type of interaction between the lines of the texture that requires different listening strategies by the musician.

Similarly, the metric structure of an isolated instrumental part contributes to the metric structure of the entire piece. Hence, it is an important question to describe different types of interaction between those parts, such as cases where different parts create metric conflicts between each other or complement each other. A listener might also follow different lines within the texture, such as different parts of a fugue requiring the description of the interaction between these parts. This article gives some examples for the investigation of these questions by analyzing isolated lines within the texture of the compositions.

Inner Metric Analysis attempts to model the invariant features of a musical piece that do not depend on a specific performance of this piece, such as tempo variations as investigated by Large & Palmer (2002). Every performer can provide additional metric information about the piece by choosing different performance strategies, such as accentuation or tempo rubato. Hence, two performances of the same piece can lead to very different metric interpretations by the listeners depending on the characteristics of the specific performance. In this article we refer to the fact, that listening to, for instance, any performance of Webern's op. 27 mvmt. 2 is very different from any performance of Joplin's Nonpareil Rag. Hence, we attempt to address these invariant features of a musical piece that do not depend on a specific performance. Therefore, symbolic data (MIDI) that represents the score of the piece is chosen as input; it guarantees the highest accuracy in the representation of these invariant features in contrast to audio data.

1.5 Methodological background and evaluation of Inner Metric Analysis

Computational models for meter perception use a wide range of different methods, such as probabilistic models (Raphael, 2001), rule-based approaches (Steedman, 1977; Longuet-Higgins & Lee, 1987; Povel & Essens, 1985; Temperley, 2001), oscillator models (McAuley, 1994; Large & Kolen, 1994; Large, 2000; Eck, 2002), linear signal processing (Scheirer, 1998) or multiagent models (Goto & Muraoka, 1999; Dixon, 2001). The methods of Inner Metric Analysis are rooted in Mathematical Music Theory, a new research direction starting in the late seventies and early eighties of the 20th century following the works of Clough (1979), Lewin (1987) and Mazzola (1985).[2] A geometrical description of the global structure of musical pieces by decomposing it into local objects and the study of their mutual relations in terms of *simplicial complexes*[3] has been introduced by Mazzola (1985). It has been applied to different analytic perspectives, such as to motivic analysis (see Nestke, 2004) and underlies Inner Metric Analysis as well. The local objects in the metric model are the pulses, which are therefore called *local meters*. This article does not discuss the geometric motivation underlying the model, for the description of the *metric complex* as a special case of a simplicial complex we refer to Fleischer et al. (2000) and Fleischer (2003).

The methods of computational models differ in their approach to the amount of metric information considered at the moment of assigning the metric interpretation to a given event. The application of Inner Metric Analysis in this article uses the full amount of information in a non-process method in order to fully explore the richness of a structural approach to meter in music. This method is hence similar to the non-process approaches applied by Povel & Essens (1985), Eck (2001) and Toiviainen & Eerola (2004). Other applications choose a process character that simulates the listener's perspective of temporally incoming information and adjusting metric interpretation, such as Longuet-Higgins & Lee (1987), Large & Kolen (1994), Large (2000), and Eck (2002).

The process method best describes the situation of a listener not familiar with the piece yet, such as in oscillator models, where each oscillator needs some time to entrain to the pulses of the pieces. Hence, at events of significant structural changes the system needs some time to adjust to the new situation. This is an appropriate model to describe a listener's perspective who needs some time for orientation in cases of changes.

However, there are other situations where this approach is not appropriate. A typical example is the perception of musical events by an ensemble player who has to coordinate his actions to other musicians. In these situations there is no time for transient states. The ability to integrate the musician's own intentions to those of his colleagues while playing requires a far more complex orientation within the structure of the piece than considering only events of the past. Complex listening strategies include information about future events which also applies to a listener's situation in the audience who is familiar with the piece. Moreover, these situations require the ability to consider both very local as well as global information about the piece. Hence models based on a process method that do not consider the maximum amount of information available about a piece of music need to be complemented by models that do take into account this sort of information.

Inner Metric Analysis has been applied within a processive approach that considers the listening process as linear through the piece and does not take events into consideration that are far away from the current event in Volk (2005b). Within this processive approach the new incoming events change the interpretation of the metric structure given so far. Thus the gradual emergence of the metric layers can be observed. However, this article applies Inner Metric Analysis in a non-process method. Hence, at every point in time both past and future information are being considered in order to describe the maximum amount of information given about the piece. The comparison of two weight types allows the comparison between

[2]Noll (2004) gives an overview over this research field.

[3]For an introduction into the terms of algebraic topology see Spanier (1966)

more locally and globally oriented information.

The evaluation of the results of computational models follows different strategies, such as the comparison to bar lines of a selected musical corpus or the comparison to data obtained in listening tests or to the results of other computational models. The diversity of models so far did not lead to a general accepted evaluation method of results.[4] An agreement on musical corpora as common test sets have not even yet been achieved for relatively similar models, such as the different versions of oscillator models, as stated by Eck (2002).

The discussion of the results of Inner Metric Analysis in this article follows different directions. The application of the metric and spectral weights uses selected examples of different genres and compares the results to observations by music theorists. For an application of the metric weights on the musical corpus of all Brahms' symphonies and Bach's B Minor Mass we refer to Fleischer (2003). This article demonstrates the model's contribution to explain results of listening experiments that refer directly to perceptual questions by investigating ragtimes used in an experiment in Snyder & Krumhansl (2001). Applications of the metric weights to experiments that investigate expressive performances have been reported in Fleischer (2003) and Volk (2003). These different evaluation strategies demonstrate the model's strength to link complex music theoretic descriptions to music cognition.

2 The Model of Inner Metric Analysis

The model of *Inner Metric Analysis* goes back to the notion of *local meters* and *metric weights* as defined by Mazzola & Zahorka (1993-1995). The extensive application of this concept within music analyses and the systematic evaluation of the results caused us to introduce a concept of *inner metric structure* (Fleischer et al. 2000). Further enhancements of the theory, such as the definition of other types of weights and the definition of a concept of *Metric Coherence* led to the introduction of the model of *Inner Metric Analysis* (Nestke & Noll 2001; Fleischer, 2002; Mazzola, 2002; Fleischer & Noll, 2002; Fleischer, 2003; Volk, 2003; Volk 2004a; Volk, 2004b). This article compares the application of the metric and spectral weights addressing two types of metric information linked to different structural levels of the inner metric structure. The application of the weights to ragtimes used in a listening experiment in Snyder & Krumhansl (2001) connects these different structural levels to perceptual phenomena.

Inner Metric Analysis is based on the idea of score events that are equally distanced and form therefore a pulse layer. This idea has been applied in a similar way in experimental studies (Parncutt, 1987; Parncutt, 1994; Large, 2000; Large & Palmer, 2002; Toiviainen & Snyder, 2003) or in music theoretic approaches (Yeston, 1976; Krebs, 1999). The model defines the pulses of a piece of music solely on the base of note onsets. It does not take into account pitch and many other features that might contribute to the induction of pulses, such as changes in the harmonic and melodic domain, dynamic accents, density accents or registral accents. Although we do not underestimate the role of this information we will argue that the temporal information indicates in many cases very differentiated metric information. This observation has been stated in other approaches as well, such as in Toiviainen & Eerola (2004) where periodicity in the onset location was the most important cue for determining the meter.

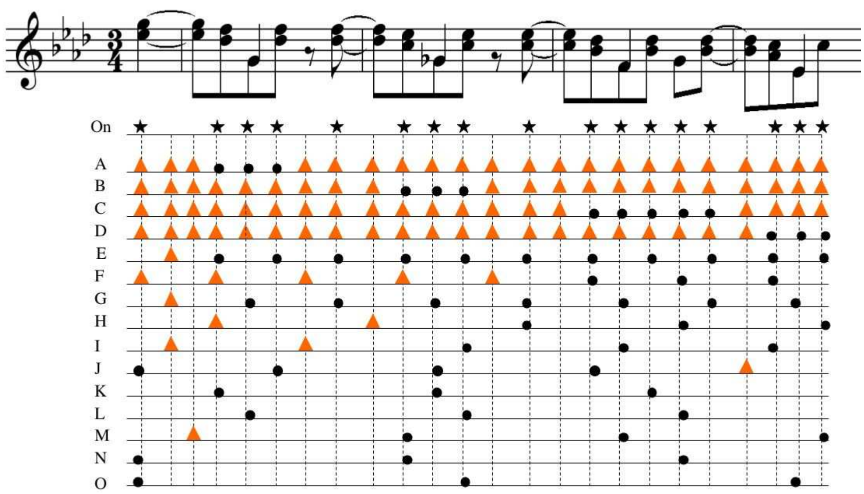

Figure 4: The Set On $\star$ and all local meters $\bullet$ and their extension $\triangle$ of the right hand from the example in Figure 3.

2.1 The Definition of the Pulses

In this section we define the specific type of pulses that underly Inner Metric Analysis. For a given piece of music we project the notes onto the set On of all onsets of notes or attack points (the point of a score where a note begins). In Figure 4 an example is shown (the right hand from the Schumann example of Figure 3) where all elements of this set are given in the first row below the notes as $\star$. By enumerating the onsets of this example as multiples of eighth notes starting with 0 for the first onset, the set On consists of the elements $0, 3, 4, 5, 7, 9, 10, 11, 13, 15, 16, 17, 18, 19, 21, 22,$ and 23. Within this set all subsets $m \subset On$ of equally spaced onsets are the candidates for the considered pulses which are called *local meters*. We consider a subset m as a local meter, if it contains at least three onsets and is maximal, hence is not a subset of any other subset $m \subset m'$ consisting of equally distanced onsets. Figure 4 shows all local meters enumerated as $A, B, C,$ ... O. The dark circles $\bullet$ indicate the local meter, the triangles $\triangle$ illustrate the extension of the local meter throughout the entire piece to be discussed in the next section. The condition of maximality can be easily illustrated by means of the local meter C. This local meter consists of the onsets $15, 16, 17, 18,$ and 19, in other words $C = \{15, 16, 17, 18, 19\}$. It contains the subset $C' = \{15, 17, 19\}$ of equally distanced onsets which is therefore not maximal and hence not considered as a local meter.

Each local meter can be identified with three parameters: the starting point or first onset s; the distance d between the consecutive onsets of the local meter (the period); and the number of repetitions k of the period which equals the number of onsets the local meter consists of minus 1, called the length. For instance, the local meter C starts at point $s = 15$, has a period of $d = 1$, consists of five onsets and has hence a length of $k = 4$. Formally we can denote any local meter as $m_{s,d,k} = \{s + id, i = 0, \ldots, k\}$, hence $C = m_{15,1,5}$. In Table 1 all local meters of the example are listed with their corresponding parameters s, d and k. The listed phase ph of a local meter is calculated as $ph = s \, modulo \, d$.

The difference between the local meters and the pulses of Figure 1 is that according to our condition the two lower rows of Figure 1 are both subsets of the uppermost row and hence

[4]For a suggestion concerning models based on symbolic data see Temperley (2004) and Volk (2005a)

do not suffice the condition of maximality.

local meter m	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
start onset s	3	9	15	21	3	15	4	13	11	0	3	4	9	0	0
period d	1	1	1	1	2	3	3	5	5	5	7	7	7	9	11
length k	2	2	4	2	10	2	6	2	2	3	2	2	2	2	2
phase ph	0	0	0	0	1	0	1	3	1	0	3	4	2	0	0

Table 1: List of all local meters of the example in Figure 4 with their corresponding parameters.

2.2 Metric weight

Based on the detection of all local meters in a given piece we define a metric weight for each onset that reflects the amount of local meters that coincide at this onset. Hence, the basic idea is similar to the explanation of metric accent patterns arising from the superposition of pulse layers according to Figure 1: points or onsets where many pulse layers coincide get a greater weight than onsets where less pulse layers coincide. Moreover, the intuition behind the metric weight is that longer repetitions should contribute more weight than shorter ones. The more stably established in a chain of successive and regular events, the more significant the onset. Hence, the weight does not simply count the number of local meters coinciding at a given onset but considers their length k. In the following we will discuss two different weights based on the local meters.

We first define the weight w_p of a local meter $m_{s,d,k}$ as the power function $w_p(m_{s,d,k}) = k^p$. Hence, instead of just considering the length k as the contribution of each local meter we use the power function k^p in order to be able to test different amounts of influence of local meters depending on their length. The higher the value of p the greater the weight of longer local meters in comparison to shorter local meters. If $p = 0$ each local meter gets the same weight $w_0 = 1$. For $p = 2$ the corresponding weights $w_2(m_{s,d,k}) = k^2$ of all local meters of our example are listed in Table 2.

The metric weight for each onset o is now calculated as the sum of weights $w_p(m_{s,d,k})$ of those local meters that inhere the onset o. The very first onset 0 in our example of Figure 4 participates in the local meters J, N and O, which all coincide at the first onset (for a complete list of the coinciding local meters for each onset see Table 3). The sum of weights of the local meters w_p in this case equals $3^p + 2 \cdot 2^p$ since the length k of J is 3 and both N and O have the length 2. We introduce a further variable parameter to our model which regulates the minimum length of the local meters denoted by ℓ. Local meters shorter than the minimum length ℓ are not considered in the calculation of the metric weight. Hence, the metric weight of a given onset o is the weighted sum of the length of all those local meters that coincide at o and have a length of a least ℓ.

In mathematical terms, let $M(\ell)$ be the set of all local meters of the piece of size at least ℓ, that is to say, $M(\ell) = \{m_{s,d,k} : k \geq \ell\}$. The general metric weight of an onset, $o \in On$, is as follows:

$$W_{\ell,p}(o) = \sum_{\{m \in M(\ell): o \in m\}} k^p. \tag{1}$$

In the case of $p = 0$ the metric weight $W_{l,0}(o)$ gives the number of local meters that coincide at the onset o, since each local meter gets the weight $w_0 = 1$. Hence, this specific case corresponds to the idea of the assignment of accents to beats depending on the number of pulses that coincide.

2.3 Spectral weight

The spectral weight is a further refinement (see Nestke & Noll, 2001) of the metric weight and is based on the extension of each local meter throughout the entire piece, denoted as $ext(m_{s,d,k}) = \{s + id, \forall i\}$ with i as integer numbers. The additional elements of each local meter in the extension in our example of Figure 4 are indicated as triangles $\triangle$ such that the extension $ext(m_{s,d,k})$ consists of all dark circles $\bullet$ and triangles $\triangle$ in one row. In the spectral weight approach each local meter contributes a weight to all events in its extension. For example, the contribution $w_p(A)$ of the first local meter A is now added to the weight of all time points of the score on a grid of eighth notes of this example, because the extension of a local meter with period $d = 1$ meets all time points of this grid. The weight $w_p(E)$ of the local meter E with period $d = 2$ contributes to the spectral weight of every other time point of this grid of eighth notes. Hence, the spectral weight allows the assignment of weights to silence events as well, in contrast to the metric weight that is defined only on note onsets.

The difference between the spectral and metric weights for the very first onset in Figure 4 is as follows. The metric weight of this onset depends on the contributions w_p of the local meters J, N and O. The spectral weight depends in addition to these local meters on the contributions of the local meters A, B, C, D and F since their extensions coincide at this first onset as well.

The spectral weight of a given onset or silence event t is the weighted sum of the length of all those local meters whose extensions coincide at t and have a length of a least ℓ. In mathematical terms, the spectral weight is defined as:

$$SW_{\ell,p}(t) = \sum_{\{m \in M(\ell): t \in ext(m)\}} k^p. \tag{2}$$

2.4 Results of the Metric and Spectral weights applied to Schumann's Walzer

In this section we discuss the results of the model applied to the Schumann example of Figure 4.

Figures 5 (a) and (b) show the metric and spectral weight profiles for the right hand of mm. 1-4 of Schumann's *Walzer* op. 124 no. 15 as given in Figure 4 in the case of $p = \ell = 2$. Hence, the minimum length ℓ does not exclude any of the local meters, since a local meter consists of at least three onsets according to our definition. The influence of the local meter's length on the weight is quadratic. The higher the line, the greater the corresponding weight, the background indicates the bar lines.

The corresponding actual numbers for the metric weight are given in the lowermost row of Table 3, while the second row of this table lists for each onset the local meters that coincide at this onset. Table 2 lists the weights $w_2(m)$ for all local meters m, hence the metric weight $W_{2,2}(o)$ can be easily calculated with the help of these tables. For instance, at the first onset 0 coincide the local meters J, N and O, according to the second row of Table 2 their respective weight m_2 is 9, 4 and 4 and hence the metric weight $W_{2,2}(0)$ of this onset is 17.

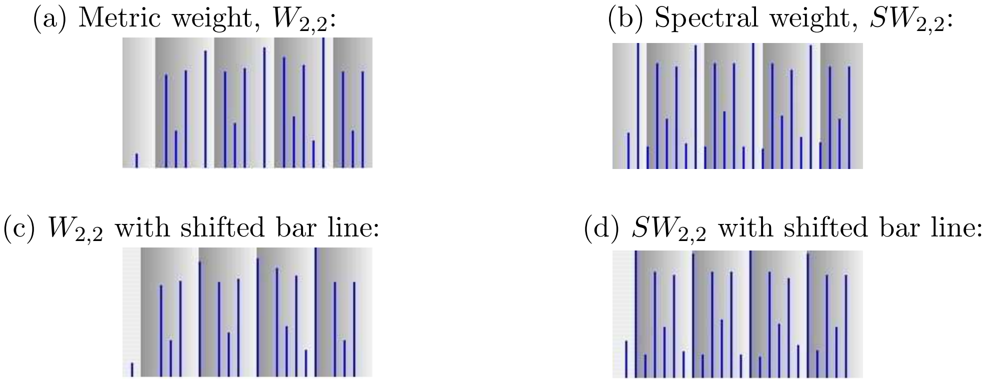

Figure 5: First Row: Metric and spectral weight profiles for the right hand of mm. 1-4 of Schumman's *Walzer* op. 124 no. 15. Second Row: corresponding weights with shifted bar lines in the background.

local meter m	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
length k	2	2	4	2	10	2	6	2	2	3	2	2	2	2	2
$w_2(m)$	4	4	16	4	100	4	36	4	4	9	4	4	4	4	4

Table 2: List of all local meters of the example in Figure 4 with their corresponding length and weight $w_2(m)$.

The metric weight profile of Figure 5(a) reveals different layers according to the height of the lines. A highest layer is built upon the weights of the sixth eighth notes in each bar in mm. 1-3 (onsets $7, 13$ and 19). A next layer is built upon the weights of the second and fourth eighth notes in each bar in mm. 1-4 (onsets $3, 5, 9, 11, 15, 17, 21$ and 23). The lowest layer corresponds to the weights of the fifth eighth note position before the first bar line, the weights of the third eighth note in bars 1, 2 and 4 and the weights of the third and fifth eighth notes in bar 3 (onsets $0, 4, 10, 16, 18$ and 22). The layers are even more distinct in the spectral weight in Figure 5(b). Here the highest layer is built upon the sixth eighth notes in all measures, followed by a layer built upon the second and fourth eighth notes of the bars. The lowest layer is built upon the weights of the first, third and fifth eighth notes of all bars.

$o \in On$	0	3	4	5	7	9	10	11	13	15	16	17	18	19	21	22	23
$m_{s,d,k}$ that meet at o	J N O	A E K	A G L	A E J	E G	B E M N	B G J K	B E I L O	E G H	C E F J	C G I M	C E K	C F H L N	C E G	D E F I	D G O	D E H M
$W_{2,2}(o)$	17	108	44	113	136	112	53	116	140	129	60	120	32	152	112	44	112

Table 3: List of all onsets o (first row) of the piece according to Figure 4 with the corresponding local meters $m_{s,d,k}$ coinciding at the respective onset (second row) and the resulting metric weight $W_{2,2}(o)$ of the respective onset (third row).

By shifting the bar lines in the background, as shown in Figures 5 (c) and (d), a correspondence between these layers and the hierarchy of the typical accent schema of a 3/4 measure becomes evident. The mentioned layers correspond to the highest layer of the first onsets of these shifted bars, followed by the second and third quarter notes, followed by the weak onsets in between. Hence, the inner metric analysis in this case reveals the characteristics of a typical 3/4 time signature, but in a shifted version. The highest layer according to the notated bar lines is built upon the upbeats of the measures.

Krebs' argument that in this piece beats other than the first beats of the bars might be perceived as the actual downbeats is hence confirmed by a phase displacement between the layers of the weights and the hierarchical layers implied by the bar lines in the analysis of the right hand.

The basic idea of a definition of metric weights based on coinciding pulses that underlies the metric and spectral weights is similar to the explanation of metric accents in music theory such as in Figure 1. However, there are essential differences between the two models. The list of the local meters in Figure 4 shows that the formation of the layers in the metric and spectral weights has its origin not in the superposition of nested pulses that have mutual dividing periods and the same phase as in Figure 1. The quarter note pulse and three quarter note pulse of Figure 1 would not be considered as local meters, because they are both subsets of the eighth note pulse and hence according to the definition of the local meters not maximal. On the other hand the metric and spectral weights realize Krebs' idea to define the meter as the union of indeed *all* layers active in the piece in the specific case of the set of the onsets of all notes.

3 A Concept of Metric Coherence

In the following we introduce the concept of *metric coherence* as the outcome of the application of the metric weight to pieces of different styles and epochs. All weights discussed are based on the parameters $p = \ell = 2$, if not indicated otherwise. Hence, no short local meters are excluded and the influence of the length of the local meters is quadratic (concerning the systematic variation of these parameters see Fleischer (2002)).

The results of the metric and spectral weights of the beginning of Schumann's *Walzer* illustrate that the metric structure described by Inner Metric Analysis may differ from the metric structure given by the bar lines. The latter one would assign the greatest accent on the silent first beat of each bar. The discrepancy between the metric structure generated by the notes and the metric structure implied by the bar lines caused Krebs to require the pianist to communicate this conflict to the audience (such as breathing on the notated downbeats). In our model we distinguish these two different structures as *inner* (generated by the notes) and *outer* metric structure (implied by the time signature and bar lines).

Within the explorative work with the metric weights (Fleischer, 2002; Fleischer, 2003; Volk 2004a; Volk, 2004b) this author has shown that in many cases a correspondence between the inner and outer metric structure can be stated. This correspondence is characterized by the existence of weight layers in the inner metric structure that reflect the hierarchical levels of the outer metric structure. Such as correspondence describes a special relationship between the two types of metric structures which we will refer to as *metric coherence* later in this section.

Figure 6 gives concerning the time signature 2/4 an example for such a correspondence.

It shows an excerpt from the analysis of the entire ragtime *Lily Queen*.[5] The highest layer is built upon the weights of the first onsets of all bars, followed by the weights of the second beats, while the second and fourth eighth notes gain much smaller weights. Hence, the weight reflects the typical hierarchical levels implied by the outer metric structure.

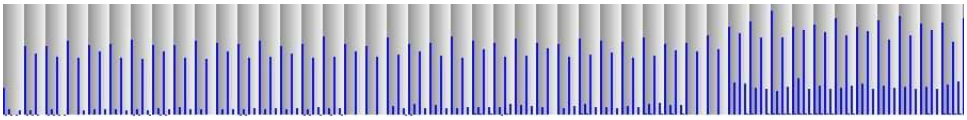

Figure 6: Excerpt from the Metric weight of the ragtime *Lily Queen* by Scott Joplin and Arthur Marshall (mm. 1 and following).

An example concerning the time signature 3/4 is given in Figure 7. It shows an excerpt of the metric weight of the entire 2nd movement of the Symphony No. 88 by Joseph Haydn. In each bar the highest metric weight is located on the first beat, while the second and third beats form a lower layer. The very low lines correspond to weights of weak eighth notes in between. Furthermore, the comparison of all weights of the first beats of the bars reveals a grouping into phrases of four bars, since every fourth bar gets a greater weight on the first onset.

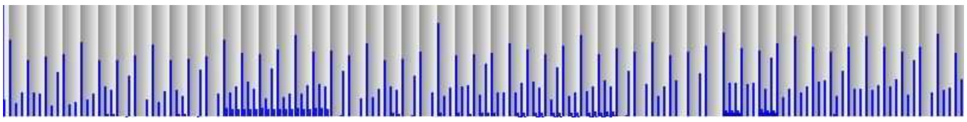

Figure 7: Excerpt from the Metric weight of Joseph Haydn's Symphony No. 88, 2nd movement (mm. 1 and following).

This grouping into a hypermeter reflects the segmentation of the piece into four bar motifs, as illustrated in Figure 8 where the opening theme in the cello is shown.

Figure 8: Theme in the cello, mm. 1-8 of Haydn's Symphony No. 88, 2nd movement.

A correspondence between the metric weight layers and the layers of the metric hierarchy implied by the bar lines (as observed in these two examples) can often be found in Renaissance madrigals which are inspired by dance rhythms, or in pieces of the Viennese classical composers. The phase shift in the metric weight observed in the Schumann example of the previous section has been found in the analyses of a number of other pieces discussed in Fleischer (2003) where the rhythmic grouping of the motives form a coherent upbeat through the piece. Hence, the inner metric structure reflects in these cases a discrepancy between the grouping of the bar lines and the grouping of the motives.

The detection of layers in the metric weight corresponding to layers of the outer metric structure resulted in the definition of a concept of *metric coherence* (see Fleischer, 2002;

[5]In this and in following examples excerpts stemming of the weight of the entire pieces are displayed in cases where the length of the entire weight would exceed the page. They are labeled as *excerpt from the weight*. In contrast to this, analyses of segments of a piece are labeled as *analyses of an excerpt*.

Fleischer, 2003). Metric coherence occurs whenever the weight layers of the metric weights reflect the layers of the outer metric structure including a layer on the bar or measure level. Hence, the existence of only two layers distinguishing between the three main beats and the weak eighth notes in between these main beats in a weight derived from a piece in 3/4 is not considered as metric coherence. The layer on the bar level (in this case the first beats of all bars) has to be part of the inner metric hierarchy. A possible quantitative measurement of metric coherence is the calculation of the correlation between the weights and templates for the different time signatures that encode the layers of the outer metric structure quantitatively (as applied for the measuring of rhythmic similarity in Chew, Volk (Fleischer), & Lee, 2005). Another possibility is the calculation of the similarity matrix[6] for the weight. An example is shown in Figure 9 for the weight from Figure 17 that reflects a 4/4 meter with eighth notes as the finest grid. Each box, say in position (i, j), represents the correlation of segment i with segment j. Dark boxes represent higher correlation. Figure 9 shows that the metric profile is repeated strongly every fourth and eighth notes, and moderately every second and sixth notes. These different correlation ranks correspond to the layers of the outer metric hierarchy. The highest correlation corresponds to the layer of the first and third quarter notes, the moderate correlation to the layer of the second and fourth quarter notes and the lowest correlation corresponds to the layer of the weak eighth notes.

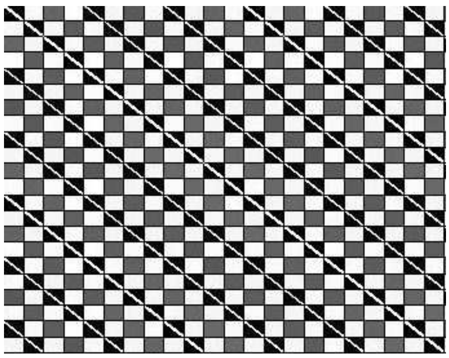

Figure 9: Excerpt from the similarity matrix for the spectral weight of Figure 17

Metric coherence describes the fact that the notes of the piece generate a metric structure that is related to the metric structure given by the bar lines. As mentioned in the introduction such a relation is not always the case. Volk (2004b) discusses examples from Brahms' symphonies where the composer creates appealing conflicts between the inner and outer metric structures such that the layers of the weights corresponded to a different meter than the notated one. Furthermore, the weights very often assign peaks to the second and fourth main beats in a 4/4 instead of the first and third, being discussed as a typical feature of Brahms' music by Frisch (1990). On the other hand analyses of Bach's B Minor Mass in Fleischer & Noll (2002) and Fleischer (2003) reveal many examples with no or only very few weight layers, which do not inhere a layer on the measure level. This reflects the fact that Bach's music very often does not follow a strict metric schema or metricity.

One example to illustrate a discrepancy between the inner and outer metric structures is the last movement of Haydn's Sonata Op. 39. The grouping structure of the right hand (see Figure 10) does not respect the grouping into bars of 3 quarter notes as suggested by the bar

[6]See Foote et al. (2002) for the use of similarity matrices for the description of rhythmic similarity.

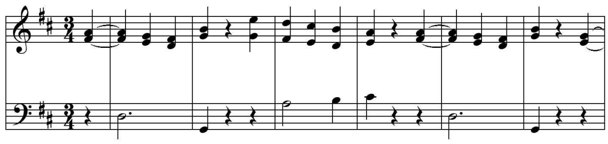

Figure 10: The beginning of Joseph Haydn's Sonata Op. 39, 3rd mvmt.

lines (time signature 3/4). The respective metric weight in the upper picture of Figure 11 reveals a periodicity that does not correspond to the periodicity of 3/4, since in many bars the highest weight is not located on the first beat.

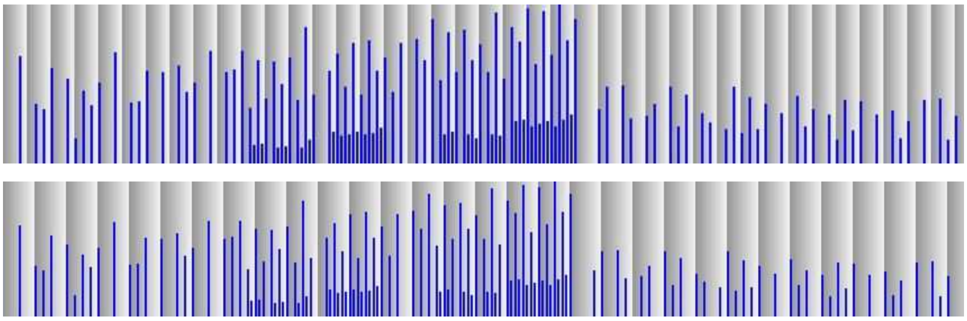

Figure 11: Metric weight of the right hand of Haydn's Sonata Op. 39, 3rd movement, mm. 1-40 with the notated bar lines according to a 3/4 (upper picture), interpreted as 4/4 (lower picture).

The periodicity suggested by the inner metric analysis becomes more evident in the lower picture of Figure 11 where the bar lines are assigned corresponding to 4/4. Now the highest metric weights in most of the bars are located on either the first or third beats, while the weights of the second and fourth beats are lower than the previous ones. Hence, the inner metric structure of the right hand seems to correspond better to a 4/4 than to a 3/4. Therefore, metric coherence does not occur. The discussion of the spectral weight of this piece in the next section of this article reveals further evidence for this assumption.[7]

Another example where metric coherence cannot be stated is the analysis of the C Major Fugue of Bach's Well-Tempered Clavier, Book I. In Figure 12 an example is shown concerning the bass voice. The superposition of all 1408 local meters results only in two layers associated with the eighth notes (higher layer) and sixteenth notes (lower layer) but in no further differentiation. Hence, metric coherence does not occur since it presupposes the occurrence of a layer on the bar level. In contrast to the Haydn example the lack of coherence is due to a lack of weight layers and not to weight layers that reveal a different periodicity than those of the bar lines.

Since all examples of this article are calculated for the parameters $p = \ell = 2$ we want to discuss at least one example with a higher value for l. Increasing the value of ℓ to 35 in the analysis of the 2nd movement of Joseph Haydn's Symphony No. 88 (see Figure 13) distinguishes the mentioned hypermeter of four bars more clearly than in the metric weight using $\ell = 2$ (as shown in Figure 7). The weights of the first beat in every fourth bar form

[7] Temperley's and Sleator's Melisma model provides a further strengthening of this assumption. The analysis of the right hand part with this model results in the assignment of a 4/4 time signature as well.

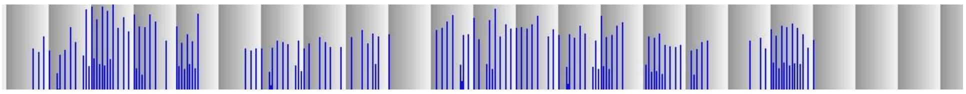

Figure 12: Metric weight of the bass of the C Major Fugue in 4/4 of Bach's Well-Tempered Clavier, Book I.

the highest layer of the weight. On the other hand the weights of some of the second or third beats tend towards 0. Hence, the first beats of all bars participate in longer local meters while the second and third main beats of the bars participate mostly in shorter local meters. For a more systematic variation of ℓ and p see Fleischer (2002) and Fleischer (2003).

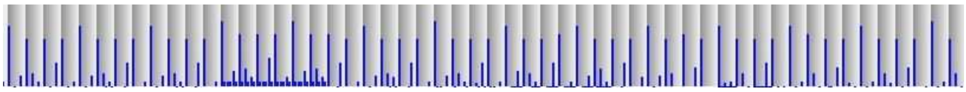

Figure 13: Excerpt from the Metric weight of Joseph Haydn's Symphony No. 88, 2nd movement, using $\ell = 35$.

The examples of this section illustrated the concept of metric coherence. Haydn's symphony demonstrated that Inner Metric Analysis is able to reveal hypermeters, as in this case the grouping into four bars. A lack of coherence is either due to the lack of weight layers (see the Bach fugue) or to weight layers that do not correspond to the outer metric structure (right hand of the Haydn sonata). Our concept of metric coherence is thus closely related to Krebs' notion of metric consonance. Both metric coherence and metric consonance characterize situations where the local meters (or pulses) align in such a way that the resulting weight pattern induced by the notes is in sink with the weight pattern induced by the outer metric structure. Furthermore, we briefly outlined the effect of a variation of the parameters ℓ and p on the results by increasing the parameter ℓ and discussing its impact onto the weight layers.

4 The Comparison of Metric versus Spectral weights

The concept of metric coherence as introduced in Fleischer (2003) is based on the evaluation of the metric weights. The more recently implemented spectral weights (see Noll, Brand, Garbers, Nestke, & Volk, 2004) are based on the extension of all local meters throughout the entire piece. This extension of the local meters may cause different effects in the spectral weight in comparison to the metric weight. Both weights hence offer a different perspective on the metric architecture of musical compositions. The metric weight conveys a more local perspective at each point of the score, the spectral weight takes more global tendencies into account and reacts more robust to local changes. The extension of the local meters in the spectral weight concept corresponds to a certain extent to an idea discussed by Krebs (1999). He argues that pulses do not only affect the metric structure in regions where they are active. Even after a pulse has stopped we may continue to count along this pulse in the following section. In the case of the spectral weights we extend the local meters also to the beginning of the piece. This extension models the effect of reinterpreting the past of a given event based on new incoming events.

As a first example to study these effects we investigate a piece that illustrates how competing local meters prevent the emergence of metric coherence. Krebs' short example stemming from Beethoven's *Eroica* in Figure 2 illustrates a superposition of pulse layers of competing periods of 2 and 3.

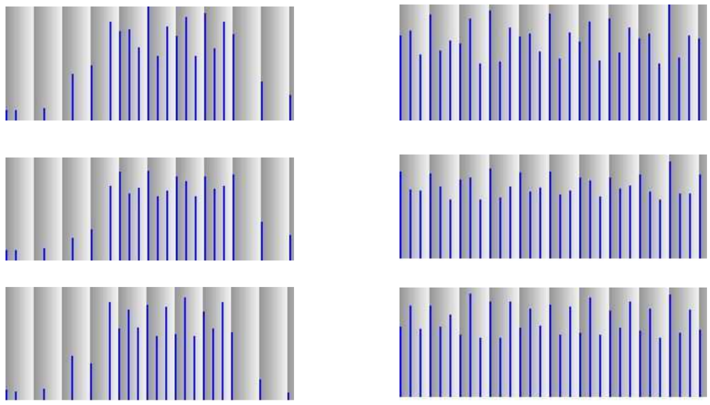

Figure 14: Upper Row: Metric (left) and spectral (right) weight profiles of mm. 681-690 of Beethoven's *Eroica*, 1st mvmt. Middle Row: The same weights as in the upper row after excluding local meters of period 2. Bottom Row: The same weights after excluding local meters of period 3.

The metric and spectral weights of this example in the upper row of Figure 14 do not reveal weight layers in the inner metric structure corresponding to the hierarchic layers implied by the time signature 3/4. But these layers evolve when we exclude all local meters of period 2 (two quarter notes) as shown in the middle row of Figure 14. Now the highest weights are located on the first beats of bars 6-9 in the metric weight (corresponding to bars 686-689 of the score). This structure is extended throughout the entire segment in the spectral weight (with the exception of the third bar). On the other hand the exclusion of all local meters of period 3 leads to a periodicity in bars 6-9 of the metric weight that corresponds to a 2/4 (see bottom row in Figure 14). Again the spectral weight continues this structure throughout the entire segment.

Hence, the competing roles of local meters of periods 3 and 2, as indicated also by Krebs in Figure 2, prevent the emergence of metric coherence in this case. A similar effect of competing periods occurs within the weights of the mentioned Haydn Sonata op. 39 when both the right and left hands are being considered in the analysis. Since the left hand "keeps" the meter of the notated 3/4 in contrast to the right hand (see Figure 11), the resulting weight profile for both hands is characterized by the superposition of competing periods (for a detailed description see Volk, 2004a). This piece hence serves as a first example addressing the metric characteristics of different lines in the texture and their interrelation. While the left hand keeps the meter, the right hand creates interesting metric conflicts in comparison to it. This phenomenon can often be observed in piano music, more examples are being discussed in section 5 concerning ragtimes.

4.1 The amplification of layers in the spectral weight

One effect that often distinguishes the spectral weight from the metric weight is the more distinct emergence of layers in the spectral weight. Figure 15 gives an example showing the spectral weight of the right hand of the Haydn Sonata op. 39. The spectral weight of the right hand reveals the distinct layers of the 4/4 meter (first and third beats as the highest layer, followed by the second and fourth beats, followed by the weak eighth notes in between) more explicitly than the metric weight given in Figure 11. Hence, the extension of the local meters results in this case in the emergence of a more distinct profile observed in the spectral weight in comparison to the metric weight. This is due to an amplification of layers. The spectral weight describes the predominant metric characteristics of this segment in the right hand as a correspondence to a 4/4 instead of the notated 3/4. Hence, metric coherence does not occur in the right hand.

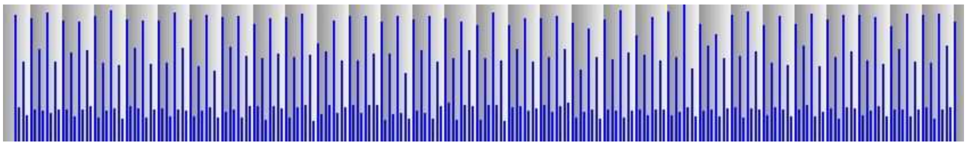

Figure 15: Spectral weight of mm. 1-40 of the right hand of Haydns Sonata op. 39, 3rd mvmt, interpreted as 4/4.

Furthermore, the spectral weight is able to amplify weak periodicities of the metric weight in cases where a piece is divided into sections that perform only a low amount of regularities in the metric weight (expressed by a low amount of weight layers). A presupposition for this effect is that the rhythmic grouping structure of the different sections of the piece does not change significantly over time. A striking example for this effect is the Fugue 4 of Bach's Well-Tempered Clavier, Part I (time signature: Alla breve). This fugue consists of five voices; each voice is composed of alternating active and silent sections.

The active segments of each voice are not characterized by different rhythmical grouping structures that would cause conflicting layers in the weight, but by different amounts of existing layers in the metric weight. An example is shown in Figure 16 concerning the bass. The first excerpt shows hardly different layers. The notes of the second excerpt generate a continuous flow in eighth notes (beginning in bar 29). In this second excerpt the four main beats of the bars create one layer and the weak eighth notes in between a lower layer. Here no layer on the bar level is present. The last excerpt shows the reentrance of the main fugue theme with great metric weights on the first and third beats of the bars, followed by those on the second and fourth beats. The weak eighth notes form the lowest layer. Hence, only the last segment generates enough layers in order to reveal a correspondence to the Alla breve time signature.

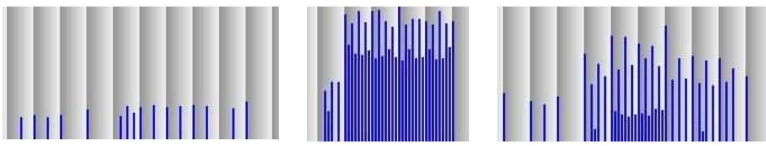

Figure 16: Excerpts from the metric weight of the bass of the Fugue 4 of Bach's Well-Tempered Clavier, Book I, bars 29-38 (left), bars 44-49 (middle), bars 73-82 (right).

In contrast to this, the spectral weight of the bass in the upper picture of Figure 17 reveals a very distinct periodicity in comparison to the metric weight. The three different layers generate metric coherence over the entire piece.

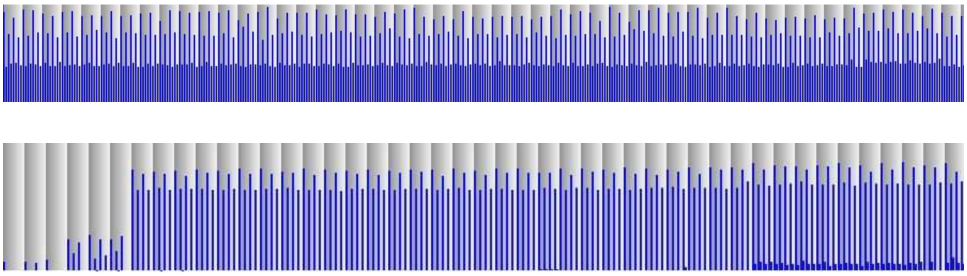

Figure 17: Above: Spectral weight of the bass of the Fugue 4 from Bach's Well-Tempered Clavier, Book I. Below: Metric weight of all voices of the Fugue 4.

Hence, in the case of this fugue the extension of the local meters connects the short segments throughout the sections of silence. Thus the surrounding context helps to establish the typical metric pattern of strong and weak beats in even those segments, where the metric weight reveals only a very little amount of differentiation.

On the other hand, this fugue is an interesting example concerning the study of polyphony with Inner Metric Analysis. The metric weight of all voices of this fugue in the lower picture of Figure 17 demonstrates that the single voices interact in such a way that the resulting structure reveals the typical layers of the Alla breve bar already from the local perspective of the metric weight. These layers are very stable throughout the entire piece. An excerpt of the metric weight of the combination of the highest and lowest voices of this fugue in Figure 18 shows an intermediate state where the layers are distinctive in some parts (such as in the last segment of the picture) but less apparent in others (such as in the beginning of the picture.) This kind of interplay of voices we called *mutual metric backing* in Fleischer & Noll (2002). It is opposed to *metric annihilation* where the interplay of voices tends towards the decreasing of coherence in comparison to the coherence of single voices as observed in other examples in Fleischer & Noll (2002). These different types of interaction between voices are especially interesting for the perception of metric structures in pieces where different instruments perform these parts. In the case of mutual metric backing it might be of great importance for the musician to know how his part combines with another part in such a way that a coherent structure arises. Knowing the structure of the combined parts might be of considerable help for the performance of his or her own part.

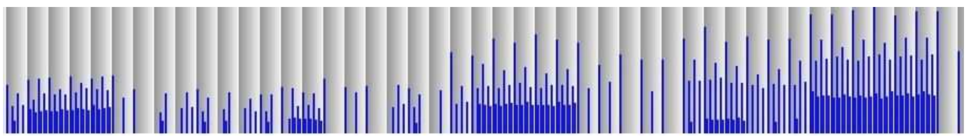

Figure 18: Excerpt from the metric weight of the bass and soprano voices of the Fugue 4 of Bach's Well-Tempered-Clavier, Book I (bars 35-79).

4.2 The sensitivity of the metric weights concerning changes of the metric state

The spectral weights are more stable and hence less sensitive to local changes of the inner metric structure in comparison to the metric weights. Depending on the rhythmical grouping structure, this behavior may cause a different effect in the spectral weights than the amplification of layers discussed in the previous sections. In the following we discuss that significant changes in the inner metric structure may not be reflected by the spectral weight but by the metric weight.

A shift in the grouping structure takes place in the melody from the first large section of the third movement of Brahms' 3. Symphony that is notated in E sharp Major. The motif that introduces a change in the grouping structure (see Figure 19) is characterized by a syncopation on the third eighth note of the 3/8-bar due to a tie to the following note on the first beat of the next bar. Bar 26 contains such a tie, bar line 27/28 is the last bar line that is not characterized by a syncopation. Starting at bar line 28/29 the grouping structure systematically changes towards this syncopation.

Figure 19: Melody of mm. 26-30 of Brahms' 3. Symphony, 3rd mvmt.

The corresponding metric weight of the theme in the upper picture of Figure 20 reflects the introduction of this rhythmic grouping starting in bar 26. In the first half of Figure 20 the highest metric weights are located on the first beats of the bars. Bars 26 and 27 start to assign higher weights on the third beats in comparison to the weights on the third beats in the previous section. Beginning with bar 28, the highest metric weights that build a very distinct layer are located on the third beats of the bars (this region is indicated by the vertical line beneath the weight). Hence, this highest layer starts at the bar line 27/28 where the grouping structure changes systematically towards the syncopation. This examples thus demonstrates an interaction between the rhythmic grouping and the inner metric structure by assigning significantly high weights on the first beats in the first part and significantly high weights on the third beats in the second part of the metric weight. For the gradual reorganization of the metric weight layers starting at the beginning of the second part of the weight along the processive approach we refer to Volk (2005b).

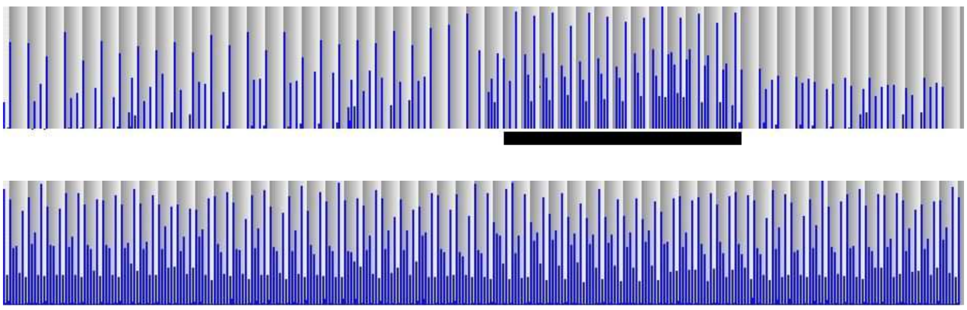

Figure 20: Above: Metric weight of the melody of mm. 1-53, 3rd mvmt. of Brahms' 3. Symphony. Below: Spectral weight of the melody of mm. 1-53.

The corresponding spectral weight in the lower picture of Figure 20 mixes these different characteristics by assigning high weights on the first and third beats through the entire segment. Thus the spectral weight illustrates the competing role of the first and third beats in the entire segment but only the metric weight is capable to assign these competing roles to different sections with a significant change of the rhythmical grouping.

The spectral weight hence might obliterate metric characteristics of adjacent segments in cases where the inner metric structure changes significantly. The metric weights of the first movement of Brahms' Fourth Symphony as discussed by Fleischer (2003) show different characteristics associated with different thematic sections in the exposition, development and recapitulation. An example is given in Figure 21 showing excerpts of the metric weight of the entire exposition concerning the wind instruments. The first excerpt (left) illustrates the metric characteristic of the first thematic section in the exposition. It is characterized by great metric weights on the second and fourth beats of the 4/4 bars. The second excerpt (right) from the same metric weight illustrates the characteristics of the second thematic section. It shows great metric weights on the third beats of all bars. The same weight layers of these thematic sections reappear in the metric weight of the recapitulation, while the development shows different characteristics. A similar effect applies to the analyses of the string instruments. This example hence illustrates an important feature of Inner Metric Analysis. Thematic sections of a sonata or symphony have often been characterized as belonging to different key regions. The metric weight allows the differentiation into different metric characterization of these segments.

Figure 21: Excerpts from the metric weight of Brahms 4. Symphony, exposition in the 1st mvmt, wind instruments.

The spectral weight of the exposition on the other hand does not change its layers throughout the piece. The extension of the local meters results in a differentiation of the layers similar to those of the second excerpt of the metric weight with high weights on the third beats, as an excerpt of the spectral weight of the exposition in Figure 22 illustrates. Hence, the different metric characteristic of the second thematic section overwrites those of the first thematic section.

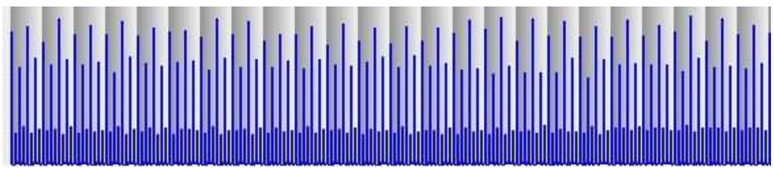

Figure 22: Excerpts from the spectral weight of Brahms 4. Symphony, 1. mvmt, wind instruments.

The comparison of the metric and spectral weights in this section has illustrated some important components of Inner Metric Analysis. Pulses of competing periods may prevent the occurrence of metric coherence. The spectral weight is able to reflect the dominant metric characteristic of a piece or segment via ignoring local changes. The study of single lines within the texture of the composition can enlighten different forms of metric interaction between those lines. The sensitivity of the metric weight reflects sudden changes and can successfully be used to distinguish the metric characteristics of the different segments.

5 Revisiting Tapping to Ragtime using Inner Metric Analysis

In the previous sections we discussed the model of Inner Metric Analysis in the spirit of a music-theoretic tool to describe the metric structure of the notes of a piece of music. In this section we apply the model to ragtimes used in a listening experiment in Snyder & Krumhansl (2001) and exemplify the potential of the model to contribute to the understanding of perceptual phenomena. The structural description of the inner metric structure of ragtimes used in the listening experiments allows a profound understanding of the experiment's results.

Listening tests using computer-generated performances as reported in Fleischer (2003) and Volk (2003) have shown that this model is relevant to our perception of metric structures. These experiments are based on the idea that a performer should mediate the structure of the piece to the listener. Hence, metric weights were used to shape timing and accentuation pattern following different performance strategies. Listeners rated these computer-generated performances on the one hand based on esthetical criteria and on the other concerning the question, as to how clearly the metric structure was expressed to them. The results indicate a relationship between analytical structures of the score (as described by the metric weights) and the understanding of the metric information by listeners.

Another application of the model to listening experiments described by Povel & Essens (1985) is reported in Fleischer (2003). The metric weights give the same results for the test set of repetitive temporal patterns as proposed by Povel & Essens (1985). In this section we will relate the results of tapping experiments in Snyder & Krumhansl (2001) to the results of Inner Metric Analysis.

Snyder & Krumhansl (2001) investigated the role of *pitch* in the perception of metric structures in the case of ragtime pieces and asked participants to tap along to different versions of these ragtimes. In order to study the influence of pitch each ragtime was presented once with pitch information and once without (all pitches were projected onto the same pitch thus producing a purely rhythmic piece).

Interestingly no significant difference was detected concerning the ability of the listeners to tap to the full-pitch version in comparison to the version with no pitch information. Hence, pitch in these examples did not serve as a major clue for meter detection. However, a significant decrease in this ability was stated after the left hand was removed and the right hand presented to the listeners.

In order to explain these experimental findings Snyder and Krumhansl investigated the isolation of different musical dimensions that might have been cues for the listeners to detect the pulse on the first and fifth sixteenth notes (the two main beats in the 2/4 bar). The authors compared mean distributions of musical events (across all ragtimes) for different dimensions and state for the *right hand* only one dimension that shows peaks within the mean distribution of events per onset. The number of notes per onset reveals moderate peaks at the strong first and fifth sixteenth notes, which might help to explain the pulse that listeners tapped to. However, another peak occurs at the seventh sixteenth note, which is in contrast to this pulse. The strongest peaks for the number of onsets for the *left hand* appear on the third and seventh sixteenth notes, which is in contrast to the fact that most listeners tapped along the first and third eighth notes. Hence the number of note onsets in this case did not prove to be an appropriate means to detect the downbeats.

Furthermore, Snyder and Krumhansl calculated autocorrelation coefficients for lags in order to isolate musical dimensions that are responsible for giving cues to the listeners as to where the downbeats are located. The analysis for the left hand indeed explains the period of the pulse that most listeners tapped to, the phase of the pulse remains an open question.

In the following we argue that Inner Metric Analysis can provide a deeper understanding of the experimental results by means of more detailed analytical perspectives on the ragtimes.

5.1 Resume of the Results using Inner Metric Analysis for the Study of Ragtimes

This section discusses the analyses of the left hand parts and the combined right and left hand parts of the ragtimes in order to explain the most prominent tapping pulse of the listeners.

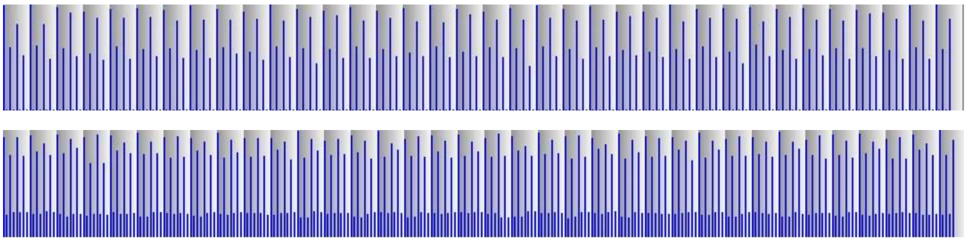

Figure 23: First Row: Spectral weight of Scott Joplin's *Nonpareil Rag* for the left hand part. Second Row: Spectral weight for both hands.

The analysis of the left hand of Scott Joplin's *Nonpareil Rag*[8] in the first row of Figure 23 shows that the *left* hand reveals the down beat pulse, on which most listeners tapped to, which are the first and second quarter notes of the 2/4 time signature. This is in contrast to the analyses of the *right* hand part (see the excerpts in Figure 25 left). The metric weight of the right hand part reveals great metric weights on the first beats of the bars, but not on the second beat. The spectral weight is characterized by an offbeat structure with high metric weights on the second, fourth, sixth and eighth sixteenth notes. However, the left hand part is characterized by metric coherence. Hence this part mediates the strong downbeats of the 2/4 bars of the ragtime. Therefore, the inner metric structure of the left hand is in this case more appropriate to explain the results than the numbers of notes per onsets. The same applies to most of the ragtimes used in the experiment, which are Scott Joplin's *Lily Queen Rag*, *Nonpareil Rag* and *Chrysanthemum Rag*, Will Nash's *Glad Cat Rag*, Raymond Birch's *Blue Goose Rag* and Joseph Lamb's *American Beauty Rag* and *Sensation Rag*. In five out of seven cases the left hand is characterized by metric coherence, examples of excerpts are given in Figure 24 concerning the *Blue Goose Rag*, the *Chrysanthemum Rag* and the *American Beauty Rag*.

Furthermore, the analysis of *both* hands of the *Nonpareil Rag* in the second row of Figure 23 results in the same layers as for the left hand. The highest weights are located on the first and third beats, the second and fourth eighth notes gain smaller weights. Further examples showing excerpts from the corresponding weights in the lower row of Figure 24 demonstrate that this is the regular case. In five out of seven ragtimes the inner metric structure of both hands reveals metric coherence. This includes the previously discussed *Lily Queen Rag* in Figure 6 (the analysis from the entire piece displayed in this figure is similar to the analysis of the segment used in the experiment).

[8]All ragtime analyses in this and the following sections concern the segments used in the second experiment in Snyder & Krumhansl (2001).

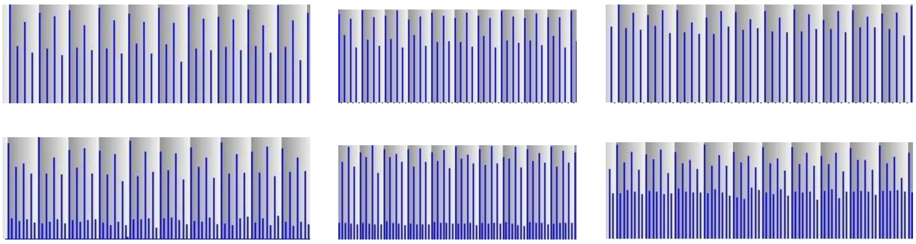

Figure 24: First Row: Excerpts from spectral weights of the left hand parts of the ragtimes *Blue Goose* (left), *Chrysanthemum* (middle) and *American Beauty* (right). Second Row: Excerpts from spectral weights of both hands of the corresponding ragtimes.

Hence, the inner metric structure of the left hand and the combined structure of both hands reveals basic clues in most of the examples as to where the downbeats are located (we will discuss the two counter-examples in section 5.3). Since this method does not consider pitch structure, these findings explain Snyder's and Krumhansl's experimental results, that no significant difference could be stated concerning the ability of the listeners to tap along the downbeats after removing the pitch information.

5.2 Syncopation in the right hand

After studying the inner metric structures of the left hand parts and the combination of both hands in the previous section we summarize in this section the findings of the analyses of the right hand parts. Presenting the right hand part only to the listeners resulted in a significant decrease of the tapping accuracy as reported in Snyder & Krumhansl (2001).

In general the isolated analyses of the right hand parts do not result in high weights on both the first and second main beats of the bar in contrast to the analyses of the left hand parts. This explains the significant difference observed in the experiments concerning the ability to tap to the downbeats while listening to the right hand only. Without any information given by the left hand the listeners ability to tap along the downbeats significantly decreased.

Moreover, Inner Metric Analysis distinguishes different amounts of syncopation present in the metric structures of the right hand of different ragtimes. The comparison of the local perspective of the metric weight to the more global perspective of the spectral weight allows to study the range of influence of the syncopation on the metric structure.

The lowest amount of syncopation among the ragtimes can be found in the *Nonpareil Rag*, *Chrysanthemum Rag*, *Blue Goose Rag* and *Lily Queen Rag*. Figure 25 shows as an example the metric and spectral weights of the *Nonpareil Rag* (left column) and those of the *Chrysanthemum Rag* (right column). The local perspective of the metric weights in the upper row of Figure 25 results in high weights on the first beats for both ragtimes. In contrast to this, the spectral weights in the lower row of Figure 25 does not show a strong first beat of the bar with a weight significantly above the other weights. Hence, from the global perspective the strength of the first beat is degraded in comparison to the local perspective. For the *Nonpareil Rag* an offbeat structure can be observed. The spectral weight of the *Chrysanthemum Rag* is characterized by high weights located on the first, third, fifth and seventh sixteenth notes indicating a less amount of syncopation in the right hand part of this piece in comparison to the *Nonpareil Rag*.

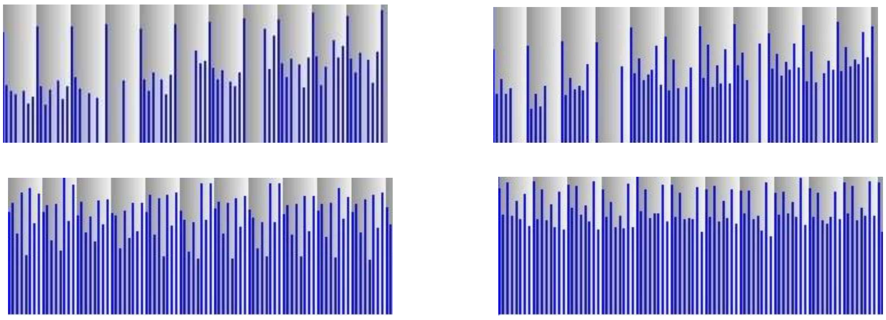

Figure 25: First Row: Excerpts from metric weights of the right hand parts of the *Nonpareil Rag* (left) and *Chrysanthemum Rag* (right). Second Row: Excerpts from spectral weights of the right hand parts of the corresponding ragtimes.

The metric structure of the right hand part of the *Sensation Rag* is similar to the structure of the *American Beauty Rag*. Even the metric weights of both pieces do not reveal high weights on the first beats of the bar, but on the offbeat as shown in the upper row of Figure 26. The spectral weights in the lower row of Figure 26 confirm this metric characteristic for both ragtimes. One can then conclude that the amount of syncopation in the right hand of the *Sensation Rag* as well as of the *American Beauty Rag* exceeds those of the *Nonpareil* or *Chrysanthemum* Rags because even the local perspective of the metric weight results in great metric weights on weak onsets with respect to the bar.

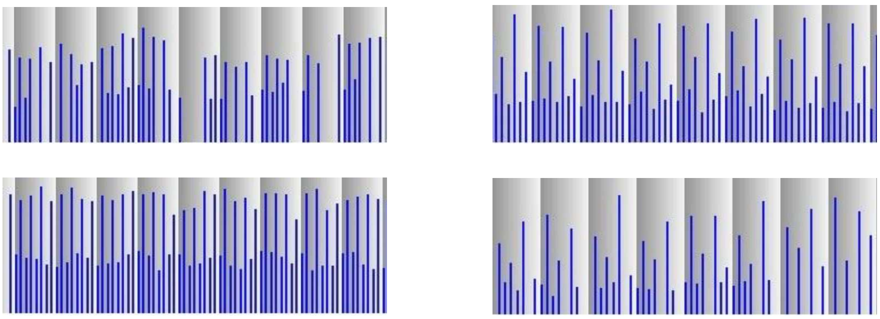

Figure 26: First Row: Excerpts from metric weights of the right hand parts of the *American Beauty Rag* (left) and *Sensation Rag* (right). Second Row: Excerpts from spectral weights of the right hand parts of the corresponding ragtimes.

The comparison of the score between, for instance, the *Nonpareil Rag* and the *American Beauty Rag* confirms that the amount of syncopation in the *American Beauty Rag* is much more exaggerated than in the *Nonpareil Rag*, as the short excerpt for the beginning in Figure 27 (right example) illustrates. The syncopations are in the *American Beauty Rag* not restricted to ties in the middle of the bar as in the *Nonpareil Rag*, as syncopations on the second and sixth sixteenth notes of the bars in the right example of Figure 27 show. They create in many bars an offbeat pattern in the right hand.

Hence, the different relations between the global and local perspectives concerning these ragtimes expressed by the metric and spectral weights respectively reflect the different amount

Figure 27: Left: Bars 5 and 6 of the *Nonpareil Rag*. Right: Bars 1 and following of the *American Beauty Rag*.

of syncopation present in the compositions. Moreover, the *American Beauty Rag* and the *Sensation Rag* resulted both in very poor listeners' performances. In each of these two pieces the analysis of the right hand part according to the comparison of the metric and spectral weights reveals a strong syncopation. This finding might contribute to an explanation of the poor tapping performances observed for these pieces.

5.3 Syncopation in the left hand

As mentioned in section 5.1, Inner Metric Analysis revealed in five out of the seven ragtimes the downbeats on the first and second beat of the bars. In this section we discuss the two ragtimes *Sensation Rag* and *Glad Cat Rag*, which are not characterized by metric coherence in the left hand.

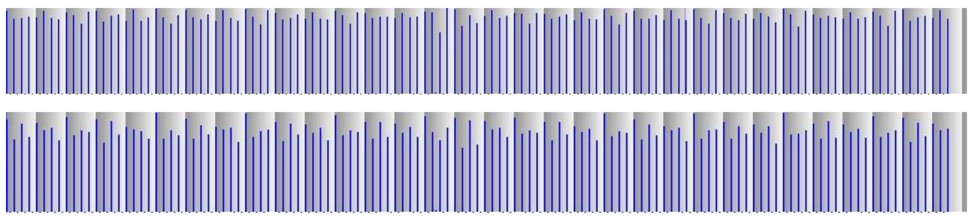

Figure 28: Above: Spectral weight of the *Sensation Rag* by Joseph Lamb for the left hand. Below: Spectral weight of the left hand after excluding two local meters.

The analysis of the left hand of the *Sensation Rag* shows no coherent metric structure, as shown in Figure 28. This fact might further contribute to the difficulty of the participants to tap on the notated downbeats of this ragtime. The lack of metric coherence in this case is due to syncopations that contribute to two local meters that are not in sink with the downbeats of the bars.

The effect of these two long local meters on the inner metric structure is demonstrated in the lower picture of Figure 28 that shows the weight after the subtraction of their contributions. Here great weights are located on the strong first and third beats in many but not all bars. This effect shows the influence of the syncopation on the inner metric structure of the left hand being responsible for preventing significantly high weights on the downbeats.

A syncopation in the left hand prevents the emergence of significantly higher weights on the downbeats in the left hand of the *Glad Cat Rag* as well. Hence, in the two ragtimes where the left hand does not reveal the highest weights on the downbeats the occurrence of syncopation is responsible for this effect.

The application of Inner Metric Analysis to ragtimes in this section helped to clarify some important observations stated in listening experiments by Snyder and Krumhansl. The metric and spectral weights do not consider pitch information but show in most of the examples metric coherence in the analysis of both hands. This explains the fact that concerning the ability of the listeners to tap to the full-pitch version of the ragtimes in comparison to the version with no pitch information the results revealed no significant difference. In addition to the autocorrelation lag used in Snyder & Krumhansl (2001) that explains the pulse that most listeners tapped to, but not the phase, the weights explain both the phase and the pulse. The isolated analysis of the left and right hand parts demonstrated that the left hand is characterized by metric coherence, while the syncopated right hand does in most of the cases not show coherence. This explains the significant difference observed in the performance of the tapping task after removing the left hand part. Furthermore, very poor tapping performances in the experiment corresponded to strong syncopation in the right hand according to Inner Metric Analysis.

6 Conclusion

The investigation of metric hierarchies induced by the notes of a piece of music is a crucial issue in both music theoretic research and in the study of music perception and cognition. This article applied the model of Inner Metric Analysis to a music theoretic description of musical pieces and to experimental findings obtained within a tapping task. Our approach demonstrates how the structural description of the metric organization of musical pieces helps to answer questions arising from cognitive perspectives and thus links both music theory and perceptual studies.

Inner Metric Analysis realizes an objective and quantitative method of creating metric weights based on pulse descriptions that arise on the notes instead of the bar lines. The model hence addresses the *inner* metric structure of a piece of music as opposed to the *outer* metric structure implied by the abstract grid of bar lines. This approach meets both crucial music theoretic and cognitive aspects of meter in music. Music theoretic studies have shown that the discrepancies between the metric state implied by the notes and the normative state of the bar lines are an important aspect of complex musical structures. Hence, Hasty (1997) demands that metric accents should not be explained only in terms of abstract grids, but as arising from the concrete elements of the piece. The definition of accents generated by the actual notes is of critical interest for the study of music perception and cognition as well. When the abstract grid of the bar lines conflicts with the inner metric structure, then the listener will have difficulty perceiving the outer metric structure unless the performer intentionally emphasizes the bar lines.

Inner Metric Analysis introduces a new reference point for perceptual metric models. The bar lines do not always reveal the most plausible metric interpretation a listener might assign to a given note sequence. Inner Metric Analysis suggests that the inner metric structure and its relation to the outer metric structure can model the structure that the listener perceives. The concept of *metric coherence* formalizes those cases, where the bar lines and the concept of the most plausible metric interpretation concur with each other. Desain & Honing (1999) evaluated Longuet-Higgins' and Lee's perceptual model by testing whether the bar lines of national anthems where assigned correctly. Inner Metric Analysis reveals that the majority of these anthems is characterized by metric coherence; hence this evaluation of a perceptual model is appropriate. Applying the model to Brahms' symphonies and comparing the results to the notated bar lines on the other hand would hardly be a successful evaluation method; Brahms' music is very often characterized by a lack of metric coherence.

Inner Metric Analysis provides a computational approach to meter that is able to address the complex descriptions of in-depth analyses of music theory. Metric coherence occurs most often in pieces that are characterized by strict metricity, such as ragtimes or Renaissance madrigals. Examples where the inner metric hierarchy according to the metric weights did not correspond to the bar lines include works by Schubert, Schumann, Beethoven, Brahms, Haydn or Webern. These examples reflect a discrepancy between the weight layers of the inner metric structure and that of the outer metric structure that has been discussed in music-theoretic literature such as Krebs (1999), Lewin (1981), Lewin (1993), Cohn (2001), Frisch (1990) or Epstein (1987). Another form of absence of metric coherence is the lack of weight layers in the inner metric weight. In these cases no regular weight pattern is evoked by the notes, such as in pieces by Dufay, Bach, or Stravinsky. Hence, metric coherence frames music theoretic findings within an objective and quantitative method.

The comparison of two different weight definitions in this article addressed different structural levels of the metric organization of a piece of music. The spectral weight addresses the predominant metric structure of a piece or segment; the metric weight reacts more sensitive to local changes. In a similar fashion, tonal studies distinguish between the main key of a piece or segment and local variations that include modulations into different keys. The comparison of the metric and spectral weights in this paper suggests that descriptions of the metric organization of a piece should include these different levels as well. The spectral weight is able to amplify weight layers that are less distinct in the metric weight. On the other hand, the metric weight grasps immediate changes of metric states and is able to distinguish between passages that are characterized by different metric states. In the case of syncopation the consideration of both the metric and spectral weights helps to distinguish between different amounts of syncopation present in a given piece.

The investigation of different types of interaction between different lines of the musical texture in this article describes an important ingredient of the metric organization of musical pieces that has not yet been addressed in detail by other applications of computational metric models. Similarly, music theoretic approaches have explored the tonal interrelation between those lines to a much greater extent than rhythmic-metric interrelations. Investigating the different lines of the musical texture with Inner Metric Analysis, such as different instrumental and vocal parts, enlightens different modes of interactions between the voices. The analyses of the interrelation between the right and left hand parts of ragtimes in section 5 shows the impact of distinguishing different lines and their contribution to the metric organization of the entire composition on listening processes.

Inner Metric Analysis relates music theoretic structures to perceptual phenomena. The re-examination of the ragtimes applied in an empirical experiment by Snyder & Krumhansl (2001) using Inner Metric Analysis allowed a detailed understanding of the listeners' ability to tap along these pieces under the different conditions. Metric coherence was found in the analyses of both hand parts. This explains the experimental findings that no significant difference could be stated concerning the accuracy of the listeners to tap along the ragtimes after removing pitch information. The results showed that in this case metric coherence did not depend on pitch information. In contrast to this, the right hand parts were not characterized by metric coherence. This results explains the significant difference in the ability to accurately tap to the piece that was stated after the left hand was removed.

The structural description of musical pieces realized by Inner Metric Analysis as discussed in this paper highlights multiple impacts on the investigation of perceptual phenomena. The comparison of local and global perspectives on the metric structure is a prerequisite for the description of complex listening strategies that involve both levels of information. The robustness of the spectral weight is comparable to a listening strategy that relates chains of equally spaced events (local meters) despite interruptions in between them, where the notes do not imply these chains. The sensitivity of the metric weight refers to a strategy where every change is noted instantly. Restricting the scope of the local meters to shorter segments in order to model a shorter span of information available to a listener at each point in time within a process method opens a different direction in the investigation of the different contributions of the metric and spectral weights to such listening strategies (see Volk, 2005b). The investigation of the interaction between different lines within the texture of the entire piece can serve as an important tool to investigate the perceptual strategies involved in ensemble playing as addressed in Fleischer & Noll (2002).

Inner Metric Analysis investigates the invariant metric features of a piece as in the notes of a score and does not consider information related to specific performances of the piece, such as tempo deviations, to generate metric hierarchies. Nevertheless, relating the analytic insights of the model to performance data enlightens important interrelations between musical analysis and performance. The performance experiments reported in Fleischer (2003) and Volk (2003) used the metric weights as an analytic tool that shapes a specific performance to relate metric information to the audience. Conversely, Beran & Mazzola (2000) followed the opposite strategy of relating the weights to performance data. They compared performances of different pianists playing Schumann's *Träumerei* to possible analytic strategies that can be described by different weight types. In both cases performance deviations are understood as specific means by which a performer chooses to highlight a certain analytic perspective on the piece. The results of the listening tests in Fleischer (2003) and Volk (2003) have shown that listeners are able to interpret these performance deviations in terms of metric information underlying the piece. Hence, relating Inner Metric Analysis to musical performances contributes to the investigation of the cognition of the metric information by listeners.

The model fails in those cases where pitch structure reveals the main metric information while the rhythm performs a continuous motion of equally spaced onsets. Music theorists consider information such as registral accents, density accents and harmonic or melodic changes as important for the creation of the metric hierarchy. However, Inner Metric Analysis does not use this information. In many experiments the rhythmic structure dominates over other factors of the composition, such as in the process of meter or beat induction. The failure of the model in those cases where the onset patterns are not sufficient can help to identify the interesting pieces where features other than the time organization of the piece reveal the metric clues. Musical styles characterized by a very distinct metric organization based on the onsets, such as dance rhythms, are suitable for highly differentiated metric investigation. They allow the successful classification between genres of the same notated meter and average tempo as shown in Chew, Volk (Fleischer), & Lee (2005).

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8 Glossary

inner metric structure: metric structure evoked by the notes inside the bar lines
outer metric structure: metric structure associated with the time signature
local meter: maximal set of equally spaced onsets
metric coherence: occurrence of weight layers in the inner metric structure that reflect the layers of the outer metric structure
p: power value for the weight of each local meter
ℓ: minimal length for local meters considered
$m \subset On$: the set m is a subset of On
$o \in On$: o is an element of On
$C = \{15, 16, 17, 18, 19\}$: the set C consists of the elements 15, 16, 17, 18 and 19
$m_{s,d,k}$: a local meter m with starting point s, period d and length k
$w_p(m_{s,d,k}) = k^p$: the weight of the local meter $m_{s,d,k}$
$W_{\ell,p}(o)$: the metric weight of the onset o depending on the parameters ℓ and p
$\forall i$: for all i
$ext(m_{s,d,k})$: extension of the local meter $m_{s,d,k}$ throughout the piece
$SW_{\ell,p}(t)$: the spectral weight of the time point t depending on the parameters ℓ and p

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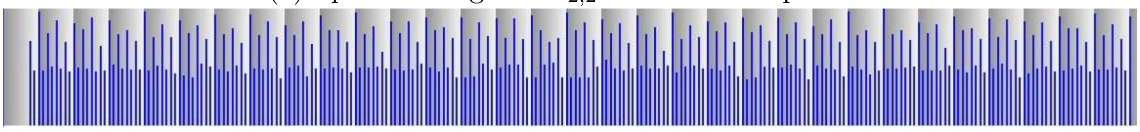
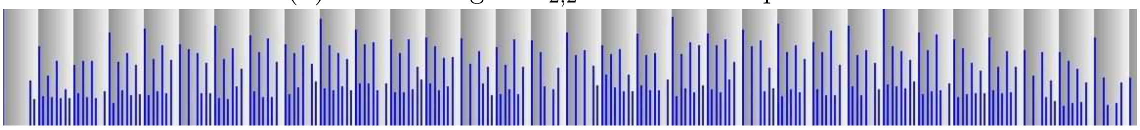
9 Appendix: Analyses of ragtime excerpts used by Snyder & Krumhansl

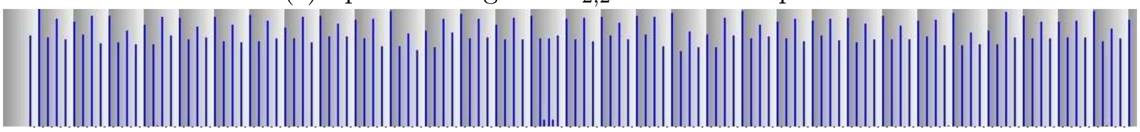
(a) Spectral weight $SW_{2,2}$ of both hand parts:



(b) Metric weight $W_{2,2}$ of both hand parts:



(c) Spectral weight $SW_{2,2}$ of left hand part:



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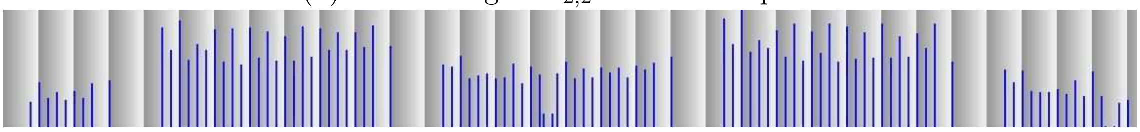
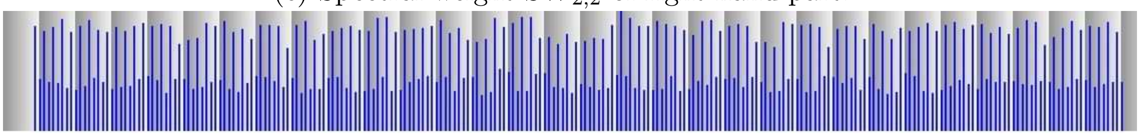


(e) Spectral weight $SW_{2,2}$ of right hand part:



(f) Metric weight $W_{2,2}$ of right hand part:

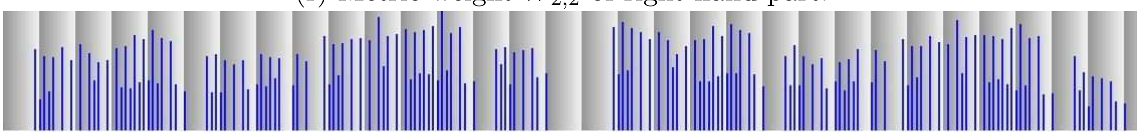
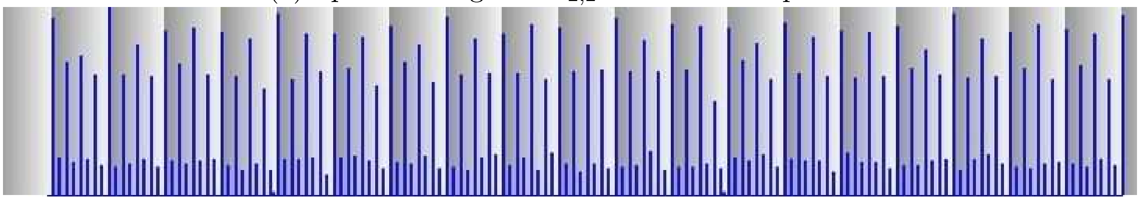
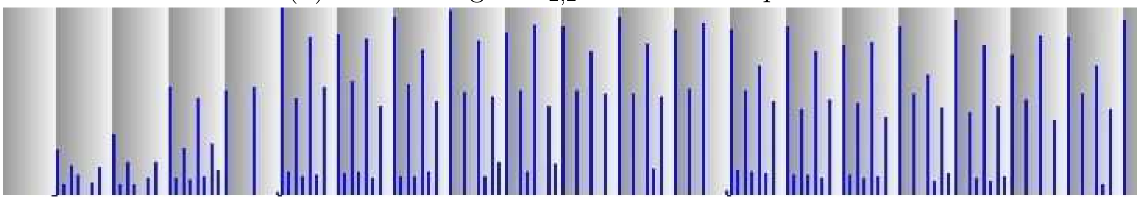


Figure 29: Analyses of American Beauty Rag

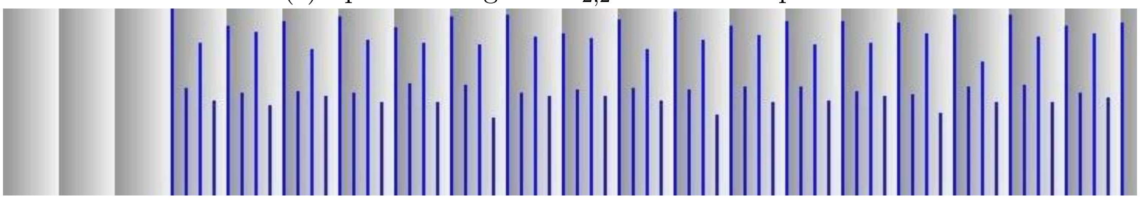
(a) Spectral weight $SW_{2,2}$ of both hand parts:



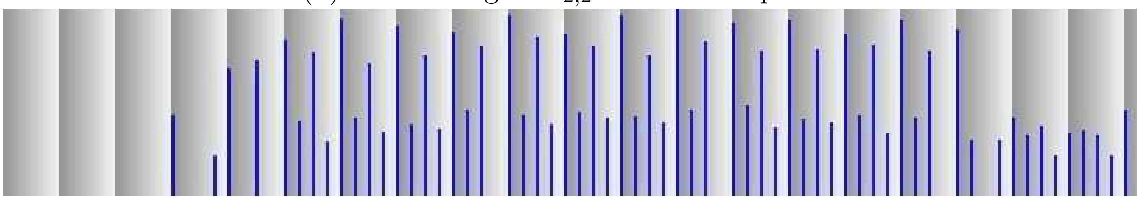
(b) Metric weight $W_{2,2}$ of both hand parts:



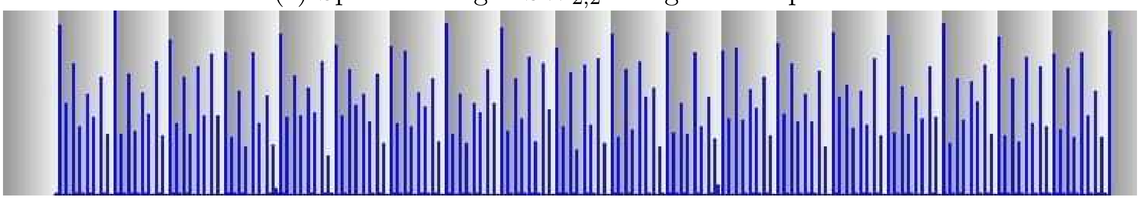
(c) Spectral weight $SW_{2,2}$ of left hand part:



(d) Metric weight $W_{2,2}$ of left hand part:



(e) Spectral weight $SW_{2,2}$ of right hand part:



(f) Metric weight $W_{2,2}$ of right hand part:

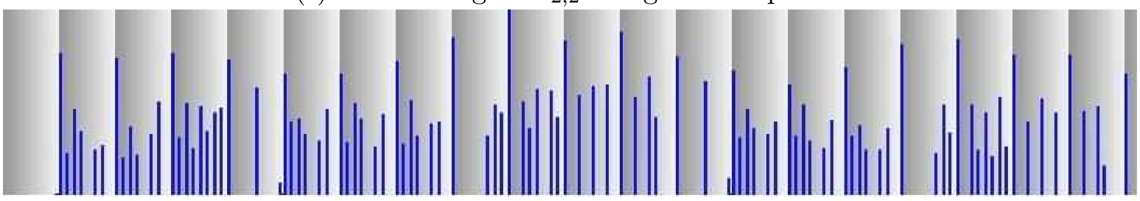
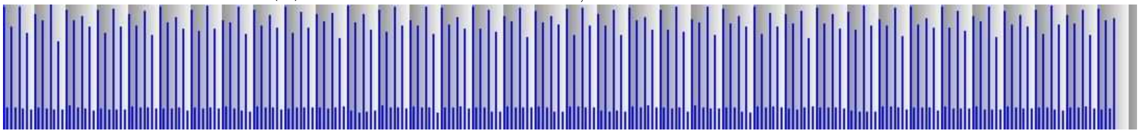
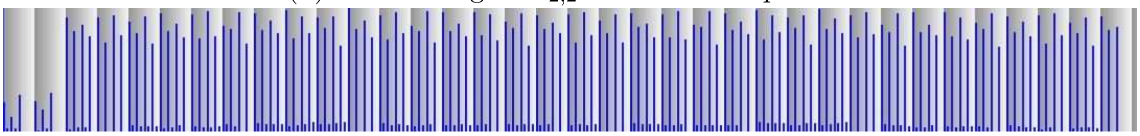


Figure 30: Analyses of Blue Goose Rag

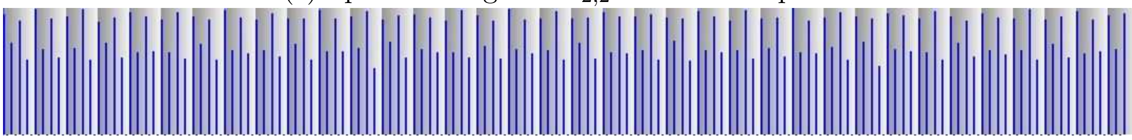
(a) Spectral weight $SW_{2,2}$ of both hand parts:



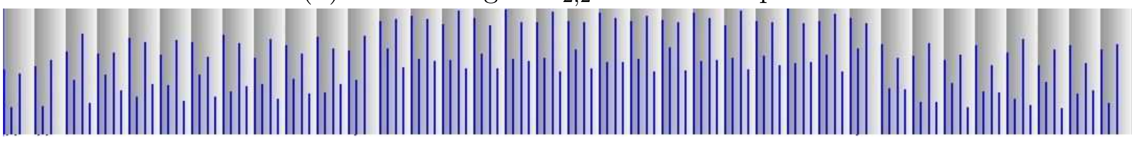
(b) Metric weight $W_{2,2}$ of both hand parts:



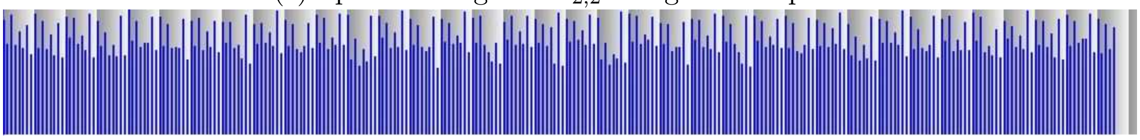
(c) Spectral weight $SW_{2,2}$ of left hand part:



(d) Metric weight $W_{2,2}$ of left hand part:



(e) Spectral weight $SW_{2,2}$ of right hand part:



(f) Metric weight $W_{2,2}$ of right hand part:

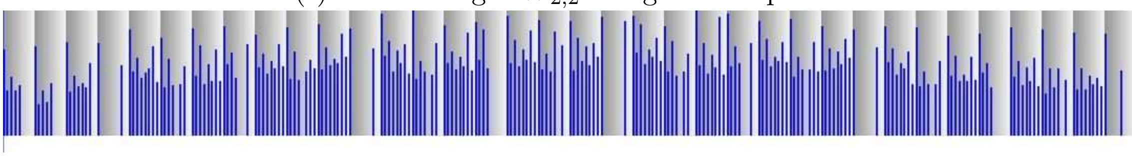
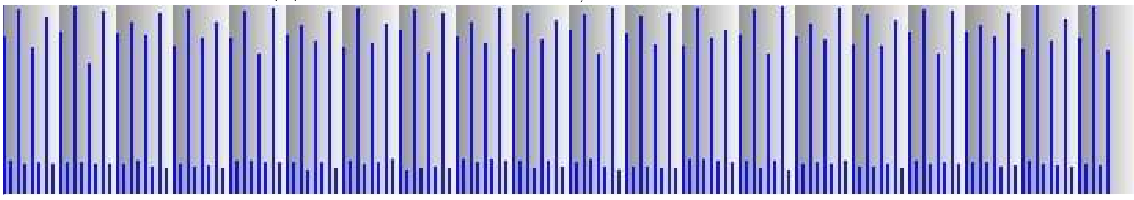
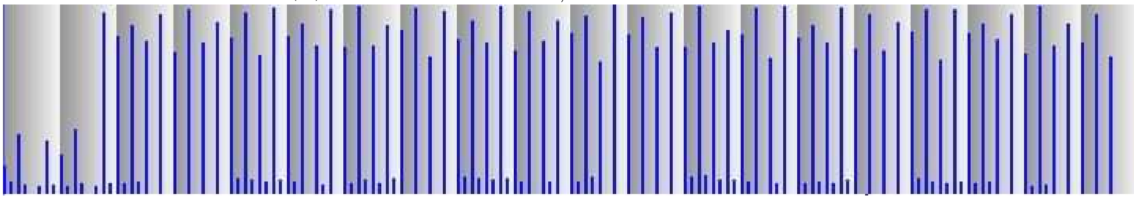


Figure 31: Analyses of Chrysanthemum Rag

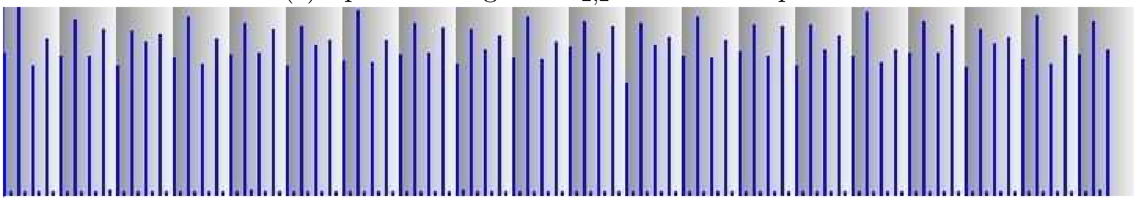
(a) Spectral weight $SW_{2,2}$ of both hand parts:



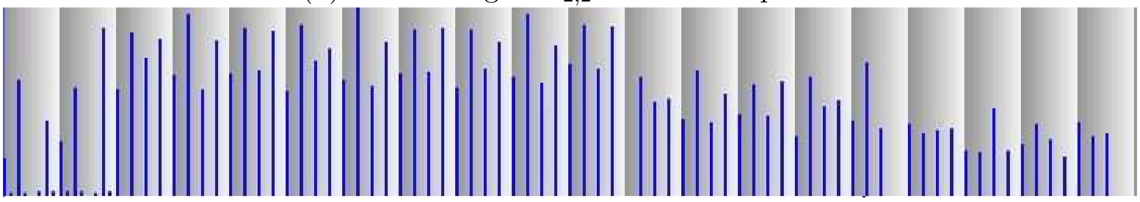
(b) Metric weight $W_{2,2}$ of both hand parts:



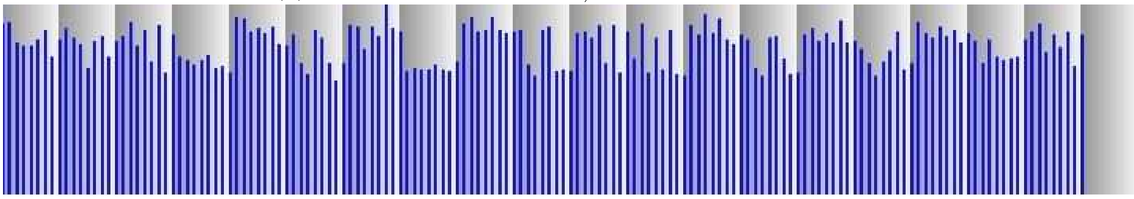
(c) Spectral weight $SW_{2,2}$ of left hand part:



(d) Metric weight $W_{2,2}$ of left hand part:



(e) Spectral weight $SW_{2,2}$ of right hand part:



(f) Metric weight $W_{2,2}$ of right hand part:

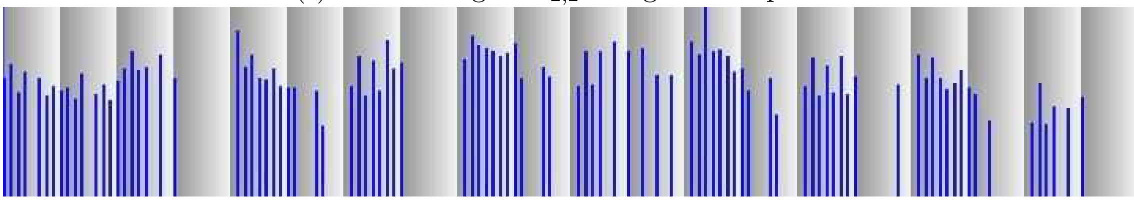
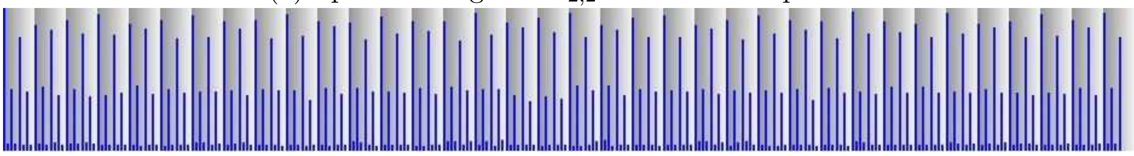
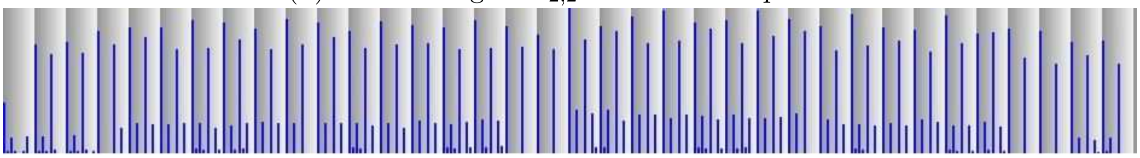


Figure 32: Analyses of Glad Cat Rag

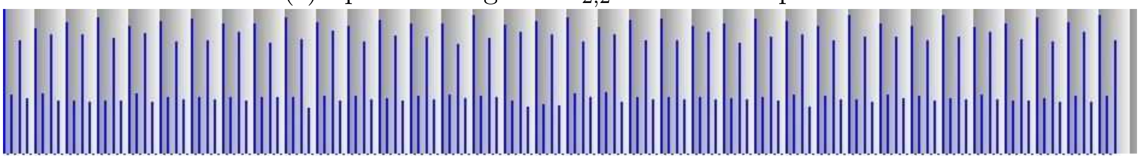
(a) Spectral weight $SW_{2,2}$ of both hand parts:



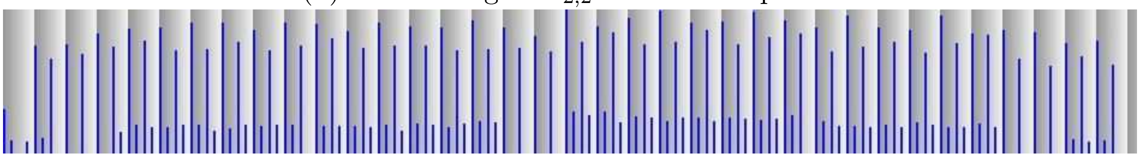
(b) Metric weight $W_{2,2}$ of both hand parts:



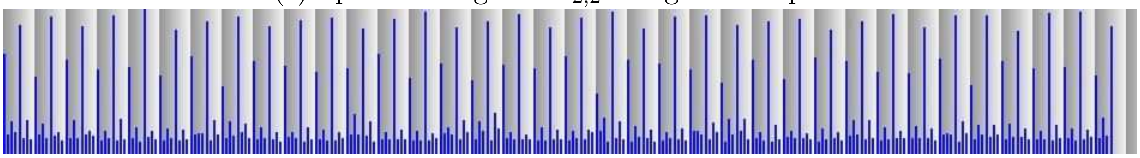
(c) Spectral weight $SW_{2,2}$ of left hand part:



(d) Metric weight $W_{2,2}$ of left hand part:



(e) Spectral weight $SW_{2,2}$ of right hand part:



(f) Metric weight $W_{2,2}$ of right hand part:

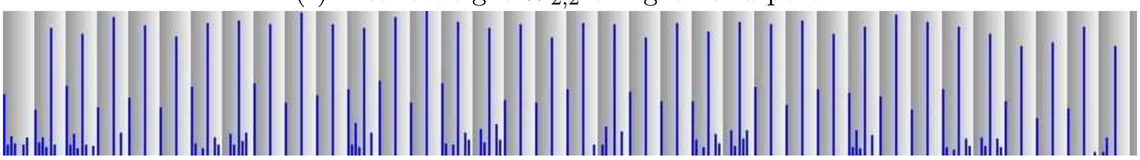
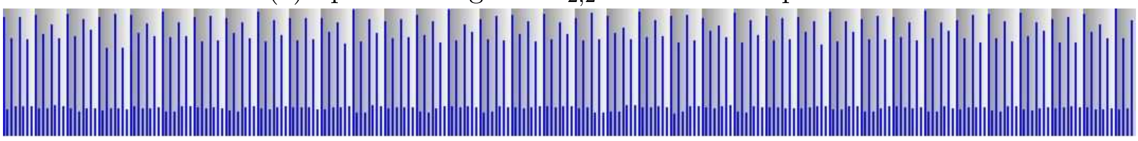
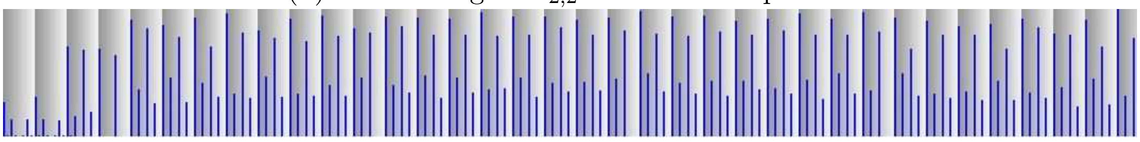


Figure 33: Analyses of Lily Queen Rag

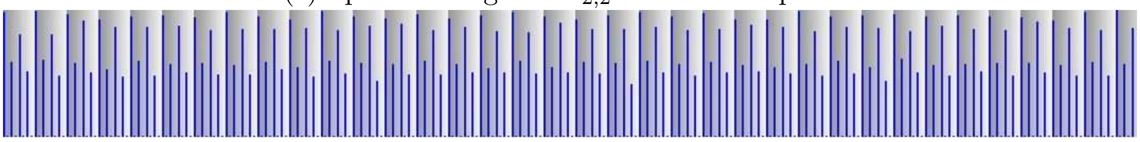
(a) Spectral weight $SW_{2,2}$ of both hand parts:



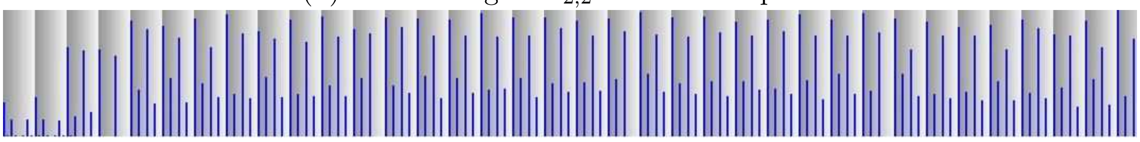
(b) Metric weight $W_{2,2}$ of both hand parts:



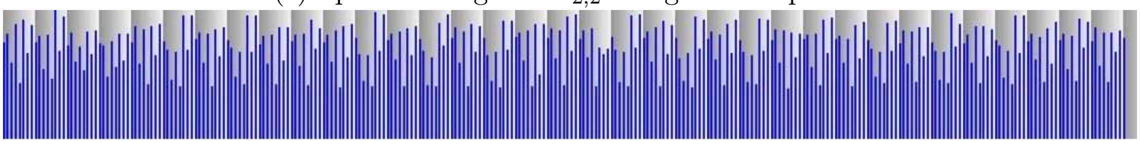
(c) Spectral weight $SW_{2,2}$ of left hand part:



(d) Metric weight $W_{2,2}$ of left hand part:



(e) Spectral weight $SW_{2,2}$ of right hand part:



(f) Metric weight $W_{2,2}$ of right hand part:

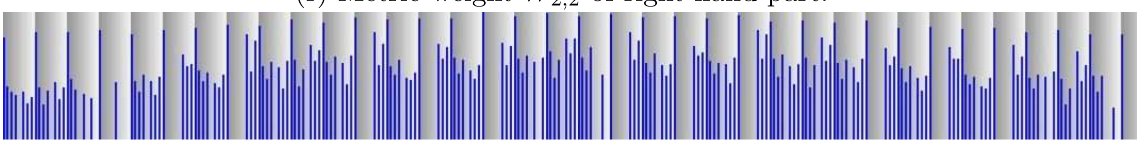
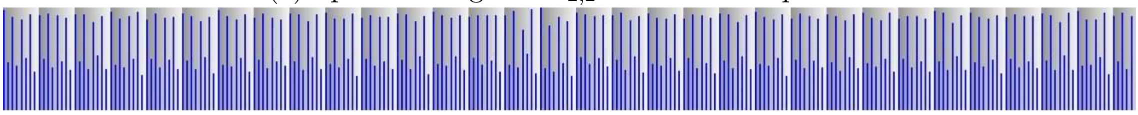
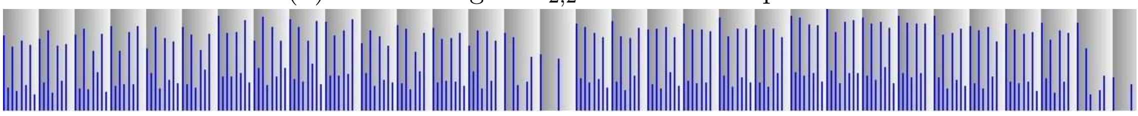


Figure 34: Analyses of Nonpareil Rag

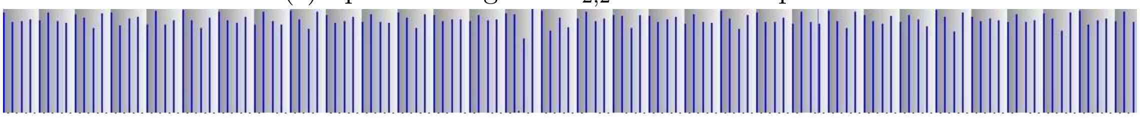
(a) Spectral weight $SW_{2,2}$ of both hand parts:



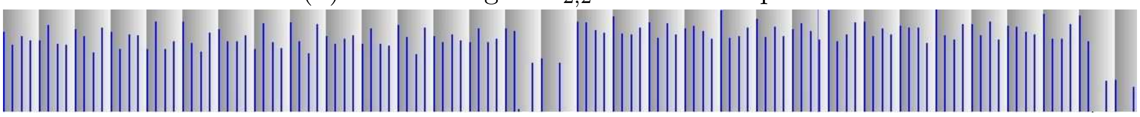
(b) Metric weight $W_{2,2}$ of both hand parts:



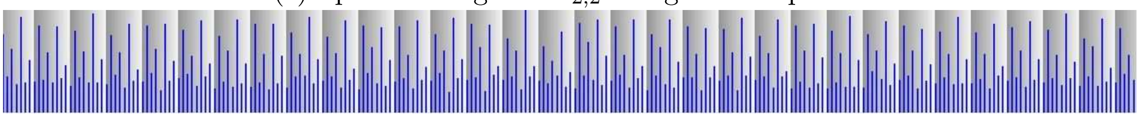
(c) Spectral weight $SW_{2,2}$ of left hand part:



(d) Metric weight $W_{2,2}$ of left hand part:



(e) Spectral weight $SW_{2,2}$ of right hand part:



(f) Metric weight $W_{2,2}$ of right hand part:

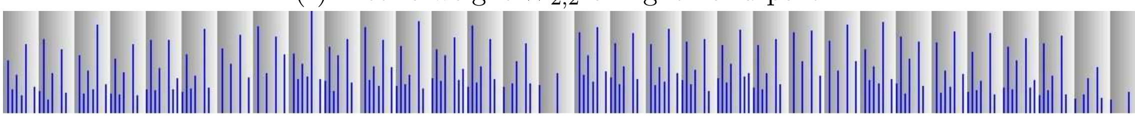


Figure 35: Analyses of Sensations Rag